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Rational Expression in Fuzzy Metric Space with Fixed Point Theorems

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ABSTRACT: Present review article deals with complete fuzzy metric space (CFMS), for picking up some fixed-point theorem results by cooperation of ration type expression. Authors have kept the previous way for gaining the main purpose.

KEYWORDS AND PHRASES: Control Function, Contractive Condition, Complete Fuzzy Metric Space, Fixed Point Theorem, Rational Expression.

AMS (2010) SUBJECT CLASSIFICATIONS: Primary 54H25, Secondary 47H10.

1. INTRODUCTION AND PRELIMINARIES:

Sayyed [15] notified Fuzzy metric space and sequel of common fixed-point theorem using property E.A. which was the enhancement of papers of many researchers and authors whose developed the theory of fuzzy sets and its applications. The initiating of concept of fuzzy set was in year 1965 by Zadeh [22] after that in year 1968 Zadeh [23], Gupta et.al [10], Mane and Sayyed [12], Sayyed [15]. Grabeic [5] and also some other result of literature such as Vasuki [20], Gregori and Sapena [07], Gupta and Mani [9] and EI Naschie,M.S., [3] Gupta,et.al.[10] , Soni and Shukla [18], Beg et.al.[1] and Shrivastava et.al. [17] had worked in it.

In this review we have used the definitions, lemmas and proved results namely Schweizer, [16] for basic definition of continuous triangular norm (t-norm), fuzzy metric space (FMS) for Kramosil & Michalek, [11]. Using the concept of Grabiec [5] theorem which was the extension of fixed-point theorem of Banach [1] to fuzzy metric space in sense of Kramosil and Michalek [11].

Now we give some important definitions and lemmas that are used in sequel,

Definition 1.4 (Grabiec, [5]) A sequence $\{X_n\}$ in a fuzzy metric space (X, M, *) is said to be convergent to $x \in X$ if $\lim_{t\to\infty} M(X_n, x, t) = 1 \ \forall \ t > 0$.

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Definition 1. 5 (Grabiec, [5]) A sequence $\{X_n\}$ in a fuzzy metric space (X, M, *) is called Cauchy Sequence if $\lim_{n\to\infty} (X_{n+p}, X_n, t) = 1 \ \forall\ t > 0$ and each p > 0.

Definition 1.6 (Grabiec, [5]) A fuzzy metric space (X, M, *) is said to be complete if every Cauchy sequence in X converges in X.

EXAMPLE 1.1 (Gregori et al., [6]) Let (X, d) be a bounded metric space with d (x, y) < k for all x, y ∈ X. Let $g : R^+ \to (k, \infty)$ be an increasing continuous function. Define a function M as

$$M(x, y, t) = 1 - \frac{d(x,y)}{g(t)},$$

Then (X, M, *) is a fuzzy metric space on X where * is a Lukasievicz t-norm, i.e, * (a, b) = max {a+b-1, 0}.

LEMMA 1.2 If there exists k ∈ (0 ,1) such that M (x, y, kt) ≥ M (x, y, t) for all x, y ∈ X and t ∈ (0, ∞), then x=y.

In our result, we define a class Φ of all mappings ξ : [0 ,1] → [0 ,1] satisfying the following conditions:

(i) ξ is increasing on [0,1], and

(ii) ξ(t) > t, ∀ t ∈ (0, 1] and ξ(t) = t if and only if t = 1.

Now in the next part we have put on a theorem for complete fuzzy metric space (CFMS), for picking up some fixed-point theorem results by cooperation of ration type expression.

2. MAIN RESULTS

THEOREM 2.1. Let $(R, S, *)$ be a complete fuzzy metric space (CFMS) and z is a mapping In R with

$$S(za, za', bb') \geq \xi\{\lambda(a, a', b')\} \qquad \text{--- (A)}$$

Where,

$$\lambda(a, a', b') = Min\left\{S(a,a',b'), S(a,za,b'), S(a',za',b'), \frac{S(a',za'\,b')[1+S(a,za,b')]}{1+S(a,a',\ b')}, \frac{S(a,za,b')[1+S(a',\ za'b',)]}{S(a,a',\ b')}\right\} \qquad \ldots(B)$$

for all a, a' ∈ R, ξ ∈ ∅ and b∈ (0, 1), Then z has a unique fixed point.

PROOF: Set up a sequence $\{a_n\}$ in R, where a is any arbitrary point in R such that

$za_n = a_{n+1}$ for all n ∈ N.

Now professing that sequence $\{a_n\}$ is a Cauchy Sequence. Let us take $a = a_{n-1}$ and $b = a_n$ in equation (A), we get

$$S(a_n, a_{n+1}, bb') = S(za_{n-1}, za, bb') \geq \xi\{\lambda(a_{n-1}, a_n, b')\} \qquad \text{--- (C)}$$

From equation (B), we have

$$\lambda(a_{n-1}, a_n, b') =$$

$$Min\left\{S(a_{n-1}, a_n, b'), S(a_n, za_n, b'), S(a_{n-1}, za_{n-1}, b'), \frac{S(a_{n-1}, za_{n-1}, b')[1 + S(a_n, za_n, b')]}{S(a_{n-1}, a_n, b')}, \frac{S(a_n, za_n, b')[1 + S(a_{n-1}, za_{n-1}, b')}{1 + S(a_{n-1}, a_n, b')}\right\}$$

$$= Min\left\{S(a_{n-1}, a_n, b'), S(a_n, a_{n+1}, b'), S(a_{n-1}, a_n, b'), \frac{S(a_{n-1}, a_n, b')[1 + S(a_n, a_{n+1}, b')}{S(a_{n-1}, a_n, b')}, \frac{S(a_{n-1}, a_{n+1}, b')[1 + S(a_{n-1}, a_n, b')}{1 + S(a_{n-1}, a_n, b')}\right\}$$

Or

$$= Min\{S(a_{n-1}, a_n, b'), S(a_n, a_{n+1}, b'), S(a_{n-1}, a_n, b'), S(a_{n-1}, a_n, b'), S(a_{n-1}, a_{n+1}, b')\}$$

Now if $S(a_n, a_{n+1}, b') \leq S(a_{n-1}, a_n, b')$, then from equation (C), we can simply write

$$S(a_n, a_{n+1}, bb') \geq \xi\{S(a_n, a_{n+1}, b')\} > S(a_n, a_{n+1}, b')$$

From Lemma 1.2. Now suppose,

$S(a_n, a_{n+1}, b') \geq S(a_{n-1}, a_n, b')$ Then again from equation (C),

$$S(a_n, a_{n+1}, bb') \geq \xi\{S(a_{n-1}, a_n, b')\} > S(a_{n-1}, a_n, b')$$

Now by simple induction, for all n and b' > 0, we get

$$S(a_n, a_{n+1}, bb') \geq S\left(a, a, \frac{b'}{b^{n-1}}\right) \qquad \text{--- (D)}$$

Now for any positive integer r, we have

$$S(a_n, a_{n+r}, b') \geq S\left(a_n, a_{n+1}, \frac{b;}{r}\right) * \ldots * S\left(a_{n+p-1}, a_{n+p}, \frac{b'}{r}\right)$$

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Using equation (D), we get

$S(a_n, a_{n+r}, b') \geq S\left(a_n, a_1, \frac{b\prime}{rb^n}\right) * \ \ldots \ * S\left(a, a_1, \frac{b\prime}{rb^n}\right)$

Taking $\lim_{n\to\infty}$, we get

$\lim_{n\to\infty} S(a_n, a_{n+r}, b') = 1$ … (E)

This implies $\{a_n\}$ is a Cauchy sequence, therefore there exists a point u' ∈ Rsuch that $\lim_{n\to\infty} a_n$ = u'. Declaration that u' is a fixed point of z.

Considering

$S(u', zu', b') \geq S(za_n, zu', b') * S(u', a_{n+1}, b') \geq \left\{\lambda\left(a_n, u', \frac{b\prime}{2b}\right)\right\} * S(u', a_{n+1}, b')$

…(F)

Applying equation (b), we explain that

$\lambda\left(a_n, u', \frac{b\prime}{2b}\right) =$

$$Min\left\{S\left(a_n, u', \frac{b'}{2b}\right), S\left(a_n, za_n, \frac{b'}{2b}\right), S\left(u', zu', \frac{b'}{2b}\right), \frac{S\left(u\prime, zu', \frac{b'}{2b}\right)\left[1+S\left(a_n, za_n, \frac{b'}{2b}\right)\right]}{1+S\left(a_n, u', \frac{b'}{2b}\right)}, \frac{S\left(a_n, za_n, \frac{b\prime}{2b}\right)\left[1+S\left(u', zu', \frac{b'}{2b}\right)\right]}{S\left(a_n, u\prime, \frac{b\prime}{2b}\right)}\right\}$$

Taking $\lim_{n\to\infty}$ in above, we get

=

$$Min\left\{S\left(u', u', \frac{b\prime}{2b}\right), S\left(u', zu', \frac{b\prime}{2b}\right), S\left(u', zu', \frac{b\prime}{2b}\right), \frac{S\left(u\prime, zu\prime, \frac{b\prime}{2b}\right)\left[1+s\left(u', zu', \frac{b'}{2b}\right)\right]}{M\left(u\prime, u\prime, \frac{b\prime}{2b}\right)}, \frac{S\left(u\prime, zu\prime, \frac{b;}{2b}\right)\left[1+S\left(u', zu', \frac{b'}{2b}\right)\right]}{M\left(u\prime, u\prime, \frac{b\prime}{2b}\right)}\right\}$$

$= S\left(u', zu', \frac{b\prime}{2b}\right)$

Hence from equation (F), we get

$S(u', zu', b') \geq \xi\left\{S\left(u', zu', \frac{b\prime}{2b}\right)\right\} * s(a_{n+1}, u', b') > S\left(u', zu', \frac{b\prime}{2b}\right) * S(a_{n+1}, u', b')$

…(G)

Taking $\lim_{n\to\infty}$ in equation (G) and using Lemma 1.2, we get

zu' = u'.

For proving its uniqueness we have to show that u' is a unique fixed point. Taking reverse that u' is not a unique fixed point then taking another point z' in R such that zz' = z' . Supposing

$$1 \geq S(z',u',b') = S(zz',zu',b') \geq \xi\left\{\lambda\left(z',u',\frac{b\prime}{b}\right)\right\}$$
…(H)

Applying equation (B), see that

Where,

$$\lambda\left(z',u',\frac{b'}{b}\right)$$

$$= Min\left\{S\left(z',u',\frac{b'}{b}\right), S\left(z',zz',\frac{b'}{b}\right), S\left(u',zu',\frac{b'}{b}\right), \frac{S\left(u',zu',\frac{b'}{b}\right)\left[1+S\left(z',zz',\frac{b'}{b}\right)\right]}{1+S\left(z',u',\frac{b'}{b}\right)}, \frac{S\left(z',zz',\frac{b'}{b}\right)\left[1+S\left(u',zu',\frac{b'}{b}\right)\right]}{S\left(z',u',\frac{b'}{b}\right)}\right\}$$

$$= Min\left\{S\left(z',z',\frac{b'}{b}\right), S\left(u',u',\frac{b'}{b}\right), S\left(z',u',\frac{b'}{b}\right), \frac{S\left(u',u',\frac{b'}{b}\right)\left[1+S\left(z',z',\frac{b'}{b}\right)\right]}{1+S\left(z',u',\frac{b'}{b}\right)}, \frac{S\left(z',z',\frac{b'}{b}\right)\left[1+S\left(u',u',\frac{b'}{b}\right)\right]}{S\left(z',u',\frac{b'}{b}\right)}\right\}$$

This implies that either $\lambda\left(z',u',\frac{b\prime}{b}\right) = 1$ or $\lambda\left(z',u',\frac{b\prime}{b}\right) = S\left(z',u',\frac{b\prime}{b}\right)$

Using it in equation (H) we get z' = u'. Its prove that u' is a unique fixed point of z'.

REFERENCES

[1] Banach,S.(1922). sur les oprations dans les ensembles abstraits et leur application aux quations intgrales [On operations in the abstract sets and their applications to the integral equations. Fundamental mathematicae, 3, 133-181.

[2] Beg, I.,Sedghi,S. and Shobe,N., (2013), Fixed point theorems in fuzzy metric spaces, International journal of Analysis,,Vol.2013, article ID 934145.

[3] EI Naschie,M.S.,(2004) . A review of E-infinity theory and the mass spectrum of high energy particle physics. Chaos Salitons Fractals, 19, 209-236.

[4] George, A.,& Veeramani,P. (1994) . On some results in fuzzy metric spaces, Fuzzy Sets and Systems, 64 ,395-399.

[5] Grabiec,M.(1988), Fixed point in fuzzy metric spaces, Fuzzy Sets and Systems ,27,385-389.

[6] Gregori,V., Morillas,S., & Sapena, A., (2011), Examples of fuzzy metrics and applications , Fuzzy Sets and Systems , 170,95-111.

[7] Gregori,V. & Sapena ,A., (2002) , On fixed point theorems in fuzzy metric spaces, Fuzzy Sets and Systems ,125,245-252.

[8] Gupta,V.,& Mani,N.,(2014 a) ,Existence and uniqueness of fixed point in fuzzy metric spaces and its applications ,Proceeding of the second international conference on soft computing for problem solving; Advances in intelligent systems & computing , 236,217-224.

[9] Gupta,V.& Mani,N.(2014b). Common fixed points by using E.A. property in fuzzy metric spaces. Proceeding s of the Third International Conference on Soft Computing for Problem Solving: Advances in Intelligent Systems and Computing, 259,45-54.

[10] Gupta, V., Saini, R.K., Mani, N., & Tripathi, A.K. (2015). Fixed point theorems using control function in fuzzy metric spaces. Cogent Mathematics,

[11] Karmosill,I., & Michalek,J. (1975). Fuzzy metric and statistical metric spaces. Kybernetica,11,326-334.

[12] Mane, S.P., and Sayyed,S.A., , (2021).Study on common fixed points theorems in fuzzy metric spaces, Tattva-Sindhu 16 ,15-32.

[13] Saini, R.K., Gupta, V., & Singh, S.B., (2007). Fuzzy version of some fixed points theorems on expansion type maps in fuzzy metric spaces. Thai Journal of Mathematics, 5, 245-252.

[14] Saini, R.K., Kumar, M., Gupta, V., & Singh, S.B., (2008). Common coincidence points of R-weakly commuting fuzzy maps. Thai Journal of Mathematics,6, 109-115.

[15] SAYYED, S.A., (2022). Fuzzy metric space and sequel of common fixed-point theorem using property E.A. Journal of Mathematical Sciences & Computational Mathematics , Vol.3, No.2, 218-24,

[16] Schweizer, B., & Sklar, A., (1960). Statistical metric spaces. Pacific Journal of Mathematics,10, 313-334.

[17] Shrivastava,M., Qureshi,K. and Singh, A.D., (2016), Some fixed point theorems in fuzzy metric spaces, American journal of Engineering Research, vol.5 no.7, 137-141.

[18] Soni, S. and Shukla, M.K., (2018), Some fixed-point theorems in fuzzy metric space for expansion mappings, International Journal of Advanced Research in Computer Science, Vol.9, No.280-283.

[19] Subrahmanyam, P. V. (1995). A common fixed-point theorem in fuzzy metric space. Information Sciences,83, 109-112.

[20] Vasuki,R. (1998). A common fixed-point theorem in fuzzy metric space. Fuzzy Sets and Systems, 97, 395-397.

[21] Vijayaraju, P., & Sajath, Z. M. I. (2009). Some common fixed-point theorems in fuzzy metric spaces. International Journal of Mathematical Analysis, 3, 701-710.

[22] Zadeh, L. A. (1965). Fuzzy sets. Information and Control, 8, 338-353.

[23] Zadeh, L. A. (1968), Probability measures of fuzzy events, J.Math.Anal.Appl.,23,421-47