International Journal of Research and Technology (IJRT)
International Open-Access, Peer-Reviewed, Refereed, Online Journal
ISSN (Print): 2321-7510 | ISSN (Online): 2321-7529
| An ISO 9001:2015 Certified Journal |

Outgrowth of Common Fixed-Point Theorem Using Property E.A. For A Pair of Weakly Compatible Mappings in Fuzzy Metric Space

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Abstract
In this review article, we have presented outgrowth of common fixed-point theorem using property E.A. for a pair of weakly compatible maps in fuzzy metric space. Our outcome is an enhancement of previous established results.

Keywords and Phrases: E.A. Property, Weakly Compatible, Altering Distance, Fuzzy Metric Space, Fixed Point Theorem.

AMS (2010) Subject Classifications: Primary 54H25, Secondary 47H10.

1. INTRODUCTION AND PRELIMINARIES

As per known result of Zadeh [32], all researchers have extended, studied and enhanced it. Here we are giving some names of mathematician George and Veeramani[5,6] , Kramosil and Michalek [13], Grabiec[7], Fuller [4], Gregori and Sapena [8], Imdad, Ali and Hasan [9],Mihet [16], Sastry,Naidu and Krishna [21], Schweizer and Sklar [24], Romaguera ,Sapena and Tirado [20], Shirude and Aage [27], Steimann [28], Vijayaraju and Sajath [30], Jungck [11], Amari and Moutawakil [1], Mujahid Abbas [2], Sedghi,et.al.[25], Khan et.all [12], Shen,et.al.[26], Wairojjana, et.al. [31] and Manthena and Manchala [14] recently proved common fixed-point theorems in fuzzy metric space using property E.A.

Ankushrao and Sayyed [3], Jain and Sayyed [10] Mc Bratney and Odeh, [15] Mane and Sayyed [17,18] Pathak, Lopez and Verma [19] Sayyed [22, 23] Subrahmanyam [29]. also gave the theory and some important results.

2. MAIN RESULTS

Theorem 2.1. Let $(X, M, *)$ be a fuzzy metric space and S_2 , S_1 be weakly compatible self-maps of X satisfying the following property

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$$\emptyset\big(M(S_2x, S_2y, t)\big) \le \alpha_1(t)\left\{\emptyset\left\{\begin{matrix}\frac{M(S_1x, S_2x, t)M(S_1y, S_2y, t)}{M(S_1x, S_1y, t)} + \frac{M(S_1x, S_2y, t)M(S_1y, S_2x, t)}{M(S_1x, S_1y, t)} + M(S_2x, S_1x, t) \\ +M(S_1y, S_2y, t) + M(S_1x, S_1y, t)\end{matrix}\right\}\right\} + \alpha_2(t)\left(\emptyset\big(M(S_2x, S_1y, 2t)\big)\right)$$

... 2.A

Where x, y ∈ X, α_1, α_2 : (0, ∞) → (0,1), t > 0 and ∅ is an altering distance function. If S_2 and S_1 satisfy the property E.A. and the range of S_1 is a closed subspace X, then S_2 and S_1 have a unique common fixed point in X.

Proof: Suppose that S_2 and S_1 satisfy the property E.A., then there exists a sequence $\{x_n\}$ in X such thats

$$\lim_{n\to\infty} S_2x_n = \lim_{n\to\infty} S_1x_n = z \in X$$

--- 2.A'

Since $S_1(x)$ is a closed subspace of X. There exists u ∈ X such that

$$z = S_1u$$

--- 2.A"

For x = x_n, y = u, Equation 2.A becomes,

$$\emptyset\big(M(S_2x_n, S_2u, t)\big) \le \alpha_1(t)\left\{\emptyset\left\{\begin{matrix}\frac{M(S_1x_n, S_2x_n, t)M(S_1u, S_2u, t)}{M(S_1x_n, S_1u, t)} + \frac{M(S_1x_n, S_2u, t)M(S_1u, S_2x_n, t)}{M(S_1x_n, S_1u, t)} \\ + M(S_2u, S_1x_n, t) + M(S_1u, S_2u, t) + M(S_1x_n, S_1u, t)\end{matrix}\right\}\right\} + \alpha_2(t)\left(\emptyset\big(M(S_2x_n, S_1u, 2t)\big)\right)$$

Taking Limit n→∞ with using Equation 2.A' and Equation 2.A", we get

$$\emptyset\big(M(z,S_2u,t)\big) \leq \alpha_1(t)\left\{\emptyset\left\{\frac{M(z,z,t)M(z,S_2u,t)}{M(z,z,t)}+\frac{M(z,S_2u,t)M(z,z,t)}{M(z,z,t)} + M(S_2u,z,t),+M(z,S_2u,t)+M(z,z,t)\right\}\right\} + \alpha_2(t)\left(\emptyset\big(M(z,z,2t)\big)\right)$$

$$\emptyset\big(M(z,S_2u,t)\big) \leq \alpha_1(t)\{\emptyset\{M(z,S_2u,t)+M(z,S_2u,t)+M(S_2u,z,t)+M(z,S_2u,t) + \emptyset(1)\}\}+\alpha_2(t)\left(\emptyset(1)\right)$$

$\Rightarrow \emptyset\big(M(z,S_2u,t)\big) = 0$ which implies $M(z,S_2u,t)=1$ i.e. $S_2u=z$. --- 2.B

From Equation 2.A" and Equation 2.B, we have

$$S_2u = S_1u = z.$$

--- 2.B'

Since S_2, S_1 are weakly compatible, we have

$$S_2z = S_1z$$

---2. B"

Now we shall show that z is a fixed point of S_2. Suppose let us assume that $S_2z \neq z$.

In view of Equations 2.B', 2.A, with 2.B" and using properties of $\emptyset$, we get

$$\emptyset\big(M(S_2z,z,t)\big) = \emptyset\big(M(S_2z,S_2u,t)\big)$$

$$\leq \alpha_1(t)\left\{\emptyset\left\{\frac{M(S_1z,S_2z,t)M(S_1u,S_2u,t)}{M(S_1z,S_1u,t)}+\frac{M(S_1z,S_2u,t)M(S_1u,S_2z,t)}{M(S_1z,S_1u,t)}+M(S_2z,S_1z,t) +M(S_1u,S_2u,t)+M(S_1z,S_1u,t)\right\}\right\}$$

$$+\alpha_2(t)\left(\emptyset\big(M(S_2z,S_1u,2t)\big)\right)$$

$$= \alpha_1(t)\left\{\emptyset\left\{\frac{M(S_2z,S_2z,t)M(z,z,t)}{M(S_2z,z,t)}+\frac{M(S_2z,zu,t)M(z,S_2z,t)}{M(S_2z,z,t)}+M(S_2z,S_2z,t) +M(z,z,t),+M(S_2z,z,t)\right\}\right\}$$

$$+ \alpha_2(t)\ \left(\emptyset\left(M(S_2z, z, 2t\ )\right)\right)$$

$$= \alpha_1(t)\ \left\{\emptyset\left\{\frac{\emptyset(1\ ).\emptyset(1\ )}{M(S_2z, z, t\ )} + M(S_2z, z, t\ ) + \emptyset(1\ ) + \emptyset(1\ ), +M(S_2z, z, t\ )\right\}\right\}$$

$$+ \alpha_2(t)\ \left(\emptyset\left(M(S_2z, z, 2t\ )\right)\right)$$

$$= 2\alpha_1(t) + \alpha_2(t)\ \left(\emptyset\left(M(S_2z, z, 2t\ )\right)\right) < \emptyset\left(M(S_2z, z, 2t\ )\right) < \emptyset\left(M(S_2z, z, t\ )\right),\ t > 0$$

which is a contradiction. Therefore, $S_2z = z$. Thus,

$S_2z = z = S_1z$

i.e. z is a common fixed point of S_2 and S_1 ---2.C

For Uniqueness, let ω∈ X be another common fixed point of S_2 and S_1 such that

$S_2\omega = S_1\omega = \omega\ and\ \omega \neq z$

---2.C'

Then by Equations 2.C,2.C' with Equation 2.A and properties of $\emptyset$, we have

$$\emptyset\left(M(z, \omega, t\ )\right) = \emptyset\left(M(S_2z, S_2\omega, t\ )\right) \leq$$

$$\alpha_1(t)\left\{\emptyset\left\{\begin{matrix}\frac{M(S_1z,S_2z,t\ )M(S_1\omega,S_2\omega,t\ )}{M(S_1z,S_1\omega,t\ )} + \frac{M(S_1z,S_2\omega,t\ )M(S_1\omega,S_2z,t\ )}{M(S_1z,S_1\omega,t\ )} + M(S_2z, S_1z, t\ ) \\ +M(S_1\omega, S_2\omega, t\ ), +M(S_1z, S_1\omega, t\ )\end{matrix}\right\}\right\}$$

$$+ \alpha_2(t)\ \left(\emptyset\left(M(S_2z, S_1\omega, 2t\ )\right)\right)$$

$$= \alpha_1(t)\left\{\emptyset\left\{\begin{matrix}\frac{M(z, z, t\ )M(\omega, \omega, t\ )}{M(z, \omega, t\ )} + \frac{M(z, \omega, t\ )M(\omega, z, t\ )}{M(z, \omega, t\ )} + M(z, z, t\ ) \\ +M(\omega, \omega, t\ ) + M(z, \omega, t\ )\end{matrix}\right\}\right\}$$

$$+ \alpha_2(t)\ \left(\emptyset\left(M(z, \omega, 2t\ )\right)\right)$$

$$= (\alpha_1(t) + \alpha_2(t))\ \left(\emptyset\left(M(z, \omega, 2t\ )\right)\right) < \emptyset\left(M(z, \omega, 2t\ )\right) < \emptyset\left(M(z, \omega, t\ )\right),\ t > 0.$$

which is a contradiction and thus z is the unique common fixed point of S_2 and S_1.

Conclusion

The present study establishes a meaningful outgrowth of the common fixed-point theorem for a pair of weakly compatible mappings in a fuzzy metric space by employing Property E.A., thereby contributing to the ongoing development of fixed-point theory under uncertainty. The results demonstrate that the adoption of Property E.A. effectively weakens the conventional constraints of continuity, completeness, and compatibility, which are often regarded as restrictive in classical fuzzy metric frameworks. By showing that weak compatibility, when combined with Property E.A., is sufficient to ensure the existence and uniqueness of a common fixed point, the work broadens the applicability of fixed-point results to a wider class of nonlinear mappings. The fuzzy metric setting, characterized by degrees of nearness rather than crisp distances, aligns well with real-world systems involving vagueness and imprecision, and the findings reinforce the robustness of fixed-point methods in such environments. The obtained theorem generalizes several existing results in the literature as special cases, thus unifying and extending prior approaches under a single theoretical structure.

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