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A Heuristic Approach to Optimize Rental Costs in a No-Idle Two-Stage Flow Shop Scheduling Problem

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Abstract
Scheduling is an important issue for maximizing resource utilization in manufacturing. This paper deals with the no-idle two-stage flow shop scheduling problem (FSSP), from the view point of minimizing the total rental costs. The no-idle constraint, which requires continuous machine operation, is a very important constraint in real-world manufacturing systems. While classical algorithms like Johnson's Algorithm and NEH heuristic have been widely used, they usually do not take into account the optimization of the rental costs under no-idle constraints. To fill this gap, a new heuristic algorithm for finding optimum job sequence in terms of total elapsed time and total rental cost is proposed. The resulting model incorporates setup times, probabilistic processing time and job weightage to improve scheduling efficiency. A mathematical model of the problem is given, and the computational experiments were carried out for different sizes of jobs. The performance of the proposed method is compared with the well-known heuristics such as Johnson's Algorithm, Palmer's Heuristic, NEH, and Nailwal's heuristic. Experimental findings reveal that the proposed heuristic consistently outperforms conventional methods, yielding lower rental costs and improved machine utilization efficiency.

Keywords: Flow shop, setup time, no-idle constraint, optimal sequencing, scheduling optimization, rental cost minimization.

Introduction

Scheduling is an essential and fundamental activity in resource allocation in industrial systems where assets are strategically allocated to ensure the smooth execution of activities. The major objective of scheduling is to find the optimal arrangement of the operations to attain a given optimization objective. The well-known Flow Shop Scheduling Problem (FSSP) concerns the determination of the optimal sequence in which several jobs should be processed on two or more pre-established machines, in order to optimize a certain performance criterion. A significant restriction in industrial FSPs is the no idle time constraint, that is, machines cannot be idle after the process has been started. Thus, there can be no downtime and all machines have to be always on. During the past decades, numerous studies have been made on the solution of such scheduling problems. In this context, Johnson [1] developed a mathematical model that gave an optimal solution for the two-machine flow shop problem, and was a milestone in the development of scheduling theory. His work motivated a large number of researchers to study the heuristic and mathematical aspects of similar problems. In order to reduce the makespan, Palmer [2] proposed a heuristic scheme for n-job, m-machine flow shop problems. Later, Nailwal K. K. et al. proposed a heuristic approach for finding out optimal job sequence for the minimum total elapsed time, specifically when the storage between the job stage was not available. The NEH heuristic was also introduced to be successful in optimising multi-machine scheduling by minimising the total processing time [3]. Later works by Jackson [4], Ignall E. [5], Campbell, Dudek, and Smith [6], and Gupta and Shashi [7] further developed the theory and practice of flow shop scheduling, and made the foundation for current research in no-idle and cost-based scheduling models.

It is widely known that setup times are major challenges and are one of the most important complicating factors in scheduling operations. The pioneering work of Yoshida and Hitomi [8] was the start of the research for the flow shop scheduling problem in which setup times and processing times were explicitly distinguished. Building from Johnson's rule, they suggested an extended formulation where a more extensive analysis of flow shop environments could be performed. Subsequent studies, such as the model put forth by [9], used computer simulation methods with sequence-dependent setup times in the analysis and optimization of scheduling in the job shop with limited machines. This model was useful in capturing dynamic interactions among jobs sequencing and setup operations. By taking into account these factors the research led to meaningful insights into the complexities of the process involved in scheduling, and potential frameworks for better optimization strategies [10]. The no-idle flow shop scheduling problem adds some complexity, inasmuch as machines are forced to run without breaks. The first research on m-machine no idle condition in flow shops has been conducted in [11], paving the way for further research. Addressing the cost issues, Kaur et al. [12] presented an idea to minimize hiring or rental costs for no-idle two stage flow shop scheduling. Similarly, Singla et al. [13] proposed a novel approach by incorporating job weightage and transit time factors in scheduling in order to reduce leasing cost and optimize resource allocation. Inspired by nature, many modern optimization methods mimic biological and evolutionary processes to solve complex engineering problems. Scholars such as Kaur [14], Modibbo [15], and Kumari [16], [17] have successfully tried to apply these bio-inspired approaches to scheduling optimization. Furthermore, substantial contributions of Kumari S., Khurana P., Singla S., Kumar A. [18] and Malik S., Verma S., Gupta A., Sharma G., Singla S. [19] have enriched the literature with rich detailed studies on statistical and heuristic optimization techniques in flow shop scheduling and other industrial related applications.

Practical situation

In actual manufacturing and fabrication situations, different experimental and practical conditions are commonly faced. These situations often require the execution of several tasks that involve using different types of industrial equipment. The weightage of jobs can be noticed in a number of industries like cotton processing, leather, and textile production. These sectors give practical examples showing why the various job roles matter and contribute to the greater efficiency of the production process. For example, in industries that manufacture cotton cloth, leather shoes, textile clothes of various sizes and qualities, job sequencing and scheduling have to cope with various consumer preferences and market demands. Due to financial constraints at the early stages of industrial operations it is often necessary to rent rather than buy expensive machinery. For example, when a pathology laboratory is being established, several expensive devices such as microscopes, water baths, lab incubators, glucometers, blood cell counters, and the like, along with tissue diagnostic systems, are typically acquired on rent. Renting such a piece of equipment can help to save capital investments, ensure the right machine can be used for the job at hand and to have access to the latest technology without having to heavily invest in expensive technology.

Assumptions

- There is no transfer of jobs between the two machines, H1 and H2, as both operate sequentially and independently in the order H1 → H2. A single job cannot be processed simultaneously by both machines. Any change in the machine's operation path is strictly prohibited until the completion of the current job.
- Time spent on setup and equipment breakdown is not included in utilization calculations.
- Rental Policy

The machines are rented on an as-needed basis and returned once they are no longer required for production. Specifically, the first machine is acquired through a rental agreement at the beginning of job processing. After the completion of the first job on the initial machine, the second machine is then rented for subsequent operations. This approach ensures cost efficiency by minimizing idle rental periods and optimizing equipment utilization throughout the production process.

Notations

i	:	1, 2,...n sequence of jobs
s_1	:	Sequence optimization employing Johnson's method
h_{i1}	:	First machine's i-th job processing time
h_{i2}	:	Second machine's i-th job processing time
P_{i1}	:	Probability pertaining to h_{i1}
P_{i2}	:	Probability pertaining to h_{i2}
S_{i1}	:	Setup time of first machine H_1
S_{i2}	:	Setup time of second machine H_2
Q_{i1}	:	Probability pertaining to S_{i1}
Q_{i2}	:	Probability pertaining to S_{i2}
T_{i2}	:	Second machine's i-th job completion time
$u_1(s_1)$	:	The time period of machine H_1's utilization within sequence s_1
$u_2(s_1)$	:	The time period of machine H_2's utilization within sequence s_1
c_1	:	Time-based fees for rental of machine H_1
c_2	:	Time-based fees for rental of machine H_2
l_2	:	To eliminate idle time, the latest time to lease machine H_2

$r(s_1)$: Rental cost for sequence s_1

Problem Formulation

Consider the processing of jobs i (where i ranges from 1 to n) by two machines, denoted as H_1 and H_2. Take into account the processing time pertaining to probabilities P_{i1} &P_{i2} on the machines H_1 &H_2 denoted by h_{i1} and h_{i2}. Also, the setup times S_{i1} and S_{i2} pertaining to probabilities Q_{i1} &Q_{i2} on the machines H_1 and H_2 correspondingly. The model's mathematical representation can be expressed mathematically in the form of **Error! Reference source not found.**in a matrix-based format. In order to minimize capital expenditures for rented equipment, our mission is to pinpoint the optimum jobs {s_1} sequence.

TABLE 1: MATHEMATICAL FORMULATION IN A MATRIX FORMAT

Job	**Machine H₁**				**Machine H₂**			
i	h_{i1}	P_{i1}	S_{i1}	Q_{i1}	h_{i2}	P_{i2}	S_{i2}	Q_{i2}
1	h_{11}	P_{11}	S_{11}	Q_{11}	h_{12}	P_{12}	S_{12}	Q_{12}
2	h_{21}	P_{21}	S_{21}	Q_{21}	h_{22}	P_{22}	S_{22}	Q_{22}
3	h_{31}	P_{31}	S_{31}	Q_{31}	h_{32}	P_{32}	S_{32}	Q_{32}
..	..	..	..	..	..	..	..	..
n	h_{n1}	P_{n1}	S_{n1}	Q_{n1}	h_{n2}	P_{n2}	S_{n2}	Q_{n2}

Algorithm

Step 1: Determine the processing times, named as H_{i1}& H_{i2}, for the machines H_1& H_2 respectively:

$$H_{i1} = h_{i1} \times P_{i1} - S_{i2} \times Q_{i2} \quad (1)$$

$$H_{i2} = h_{i2} \times P_{i2} - S_{i1} \times Q_{i1} \quad (2)$$

Step 2: While cutting down on the total amount of time elapsed, implement on Johnson's method(1954) to acquire the optimum string s_1.

Step 3: For computing the total elapsed time for string s_1, build a flow in-out table.

Step 4: Determine

$$l_2 = T_{i2} - \sum_{n=1}^{\infty} H_{i2} \quad (3)$$

Step 5: In order for machine H_2 to commence processing, the most recent time l_2 considered as the starting point for processing will be employed to generate a flow in-flow out table.

Step 7: Calculate utilization time $u_1(s_1)$ and $u_2(s_1)$ of machines H_1& H_2 by:

$$u_1(s_1) = \sum_{n=1}^{\infty} H_{i1} \quad (4)$$

$$u_2(s_1) = T_{i2} - l_2 \quad (5)$$

Step 8: Finally, calculate

$$r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2 \quad (6)$$

Numerical Illustartion

Taking into consideration, where processing durations separating to the setup times are specified, assume five jobs and two machines. Four and six units of time are needed to hire machines H_1 and H_2, respectively. Our goal is to achieve optimal efficiency of sequencing jobs for execution on machines that may be rented for the most economical cost.

TABLE 2: PROBLEM-SPECIFIC DATA SET

Jobs	**Machine H_1**				**Machine H_2**			
i	h_{i1}	P_{i1}	S_{i1}	Q_{i1}	h_{i2}	P_{i2}	S_{i2}	Q_{i2}
1	14	0.2	4	0.2	29	0.2	5	0.1
2	29	0.2	8	0.3	31	0.1	9	0.2
3	30	0.1	6	0.2	27	0.2	4	0.3
4	9	0.3	1	0.1	5	0.3	7	0.2
5	12	0.2	3	0.2	8	0.2	2	0.2

Solution:

In accordance with Step 1, TABLE III. presents an overview of the anticipated processing times on machines H1 and H2

Table 3:: Expected Process Time On Machines

i	H_{i1}	H_{i2}
1	2.3	5.0
2	4.0	0.7
3	1.8	4.2
4	1.3	1.4
5	2.0	1.0

According to step 2 of the research procedure, the sequence s1 where the elements of this sequence are {4,3,1,5,2} is the optimal one that results in the least amount of time elapsed. As presented below, TABLE IV. represents the inflow and outflow based on Step 3, for schedule s1 in order to provide a comprehensive overview.

TABLE 4: TABLE FOR FLOW IN AND OUT OF STRING S1

i	H_1	H_2
4	0.0 – 1.3	1.3 – 2.7
3	1.3 – 3.1	3.1 – 7.3
1	3.1 – 5.4	7.3 – 12.3
5	5.4 – 7.4	12.3 – 13.3
2	7.4 – 11.4	13.3 – 14.0

Thus, total elapsed time $C_{max} = 14.0$

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As per **Step-5;** $l_2 = 14.0 - 12.3 = 1.7$
According to **Step 6** of the research methodology,
an IN-OUT table should be created to address the revised scheduling problem

TABLE 5: TABLE OF FLOW IN-OUT FOR ROUTE H1→ H2 WITH ZERO IDLE TIME

Jobs	***Machine H_1***	***Machine H_2***
	Inflow- Outflow	***Inflow- Outflow***
4	0.0 – 1.3	1.7 – 3.1
3	1.3 – 3.1	3.1 – 7.3
1	3.1 – 5.4	7.3 – 12.3
5	5.4 – 7.4	12.3 – 13.3
2	7.4 – 11.4	13.3 – 14.0

As per **Step-10;** $u_1(s_1) = 11.4$
$u_2(s_1) = 14.0 - 1.7 = 12.3$
As per **Step-11;** $r(s_1) = u_1(s_1) * c_1 + u_2(s_1) * c_2$
$= 11.4 * 4 + 12.3 * 6 = 119.4$ units
For machine route $H_1 \rightarrow H_2$ of the optimum sequence

Conclusion

In this paper, the proposed heuristic algorithm gives an optimal solution to the no-idle two-stage flow shop scheduling problem at the same time optimize the rental costs. The algorithm takes into account a number of things, such as processing time, job weightage and separated setup times. The main purpose of this investigation was to obtain the desired optimization results for different job sizes. Earlier studies mostly focused on small-sized job sets, with the number of jobs (n) ranging from 1 to 6 because of the complexity of the calculations. In contrast, the present work extends the analysis to medium-sized problems (7 <= n <= 30) and further to large-sized problems (31 <= n <= 80), so to expand the practical application of the proposed approach.

A set of computational experiments were performed to test the performance of the developed heuristic. The results of the experiments show that the presented algorithm is more efficient than the currently available heuristic methods proposed by Palmer (1985), Johnson (1954), NEH (1983) and Nailwal in terms of minimizing total elapsed time and rental cost. Furthermore, this research can be further developed in future studies by addition of more real life aspects as job blocking, the effects of machine breakdown and transportation time. Future work may also involve the use of trapezoidal fuzzy numbers to model machine processing times for improved modeling accuracy and decision-making precision

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