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Reliability Analysis of a Complex Series System with Correlated Failures and Repairs

[1]Vinay K. Tyagi, [2]Surbhi Goel

[1/2]M.M. (P.G.) College, Modinagar — 201204, India

ABSTRACT

The reliability of complex engineering systems depends not only on the performance of individual components but also on the interactions between component failures and repair activities. Traditional reliability models frequently assume that component failures occur independently and that repair operations are statistically independent. However, practical engineering systems often experience correlated failures due to common environmental conditions, shared power supplies, operational stress, maintenance practices and common-cause events. Similarly, repair activities may be interdependent because components share maintenance personnel, spare parts, repair facilities and operational resources. These dependencies can significantly influence overall system reliability and availability.

The present study examines the reliability characteristics of a complex series system incorporating correlated component failures and dependent repair mechanisms. A stochastic modelling framework is proposed using continuous-time Markov processes, conditional failure intensities and repair-rate parameters. The model considers the effects of common-cause failures, individual component failures, repair dependencies and system restoration. Particular attention is given to steady-state availability, reliability function, mean time to system failure and the influence of repair efficiency.

Keywords: System reliability, series system, correlated failures, common-cause failure, repair dependency, Markov process, system availability, failure intensity, repair rate, stochastic modelling.

1. INTRODUCTION

Reliability is one of the most important performance characteristics of modern engineering systems. It refers to the ability of a system, component or equipment to perform its intended function under specified operating conditions for a particular period. Reliability analysis is widely applied in manufacturing industries, electrical power networks, transportation systems, communication infrastructure, industrial automation, healthcare equipment and other technologically advanced environments.

The increasing complexity of engineering systems has created a greater need for accurate reliability assessment. Modern systems frequently contain several interconnected components whose simultaneous functioning is necessary for successful operation. A failure in one critical component may interrupt the functioning of the entire system. Such configurations are commonly represented through series-system reliability models.

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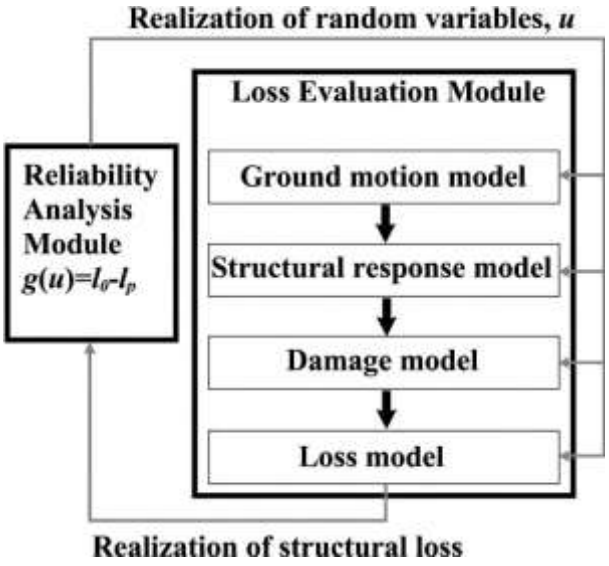


Figure: Framework for Reliability based Risk Analysis

In a conventional series system, all components must operate successfully for the complete system to remain functional. If even one essential component fails, the system becomes unavailable. For a system containing statistically independent components, reliability is generally calculated by multiplying the reliability values of individual components.

However, the assumption of independence does not always represent practical operating conditions. Components may experience similar environmental conditions, common electrical disturbances, excessive temperatures, vibration, pressure fluctuations or operational overload. These conditions can produce correlated failures, meaning that the probability of failure of one component may be influenced by the condition or failure of another component.

1.1 Background of the Study

The mathematical foundations of reliability engineering are associated with probability theory, stochastic processes and statistical life distributions.

Initially, reliability studies concentrated on individual component lifetimes and relatively simple system configurations. The exponential distribution became particularly important because it provides a convenient representation of constant failure-rate systems.

For a component having a constant failure rate ($lambda$), its reliability function is expressed as:

$$R(t) = e^{\{-\lambda t\}]}$$

where:

$R(t)$ = probability that the component operates successfully up to time (t).

lambda = constant component failure rate.

t = operating time.

Correlated failures require the introduction of additional probabilistic relationships. Similarly, repair dependency requires the consideration of repair-resource constraints and interactions between restoration processes.

1.2 Statement of the Research Problem

The central research problem concerns the inadequacy of conventional reliability models when component failures and repairs are statistically or operationally dependent.

Three major limitations can be identified:

1. Independent failure assumption: Traditional models may underestimate the probability of simultaneous component failures.
2. Independent repair assumption: Conventional approaches may overlook delays caused by shared maintenance resources.

1.3 Aim of the Study

The principal aim of the study is:

To develop and analyse a stochastic reliability model for a complex series system incorporating correlated component failures and dependent repair mechanisms.

1.4 Objectives of the Study

The objectives are:

1. To examine the reliability characteristics of complex series systems under conventional operating assumptions.
2. To investigate the influence of correlated component failures on overall system reliability.
3. To develop a mathematical framework incorporating common-cause failure mechanisms.
4. To analyse the effect of dependent repair processes on system availability.
5. To evaluate the relationship between component failure rates and repair rates.

1.5 Research Questions

The investigation is guided by the following research questions:

- How do correlated failures affect the reliability of a complex series system?
- To what extent do common-cause failures increase system unavailability?
- How does the availability of shared repair resources influence system restoration?

1.6 Research Hypotheses

The following hypotheses are proposed for analytical investigation.

Null hypotheses

- H01: Correlated component failures do not significantly influence the overall reliability of a complex series system.
- H02: Repair dependency does not significantly affect steady-state system availability.
- H03: Incorporating common-cause failures does not significantly change the estimated mean time to system failure.

1.7 Conceptual Research Framework

The conceptual framework illustrates the relationship between failure dependencies, repair mechanisms and overall system performance.

2. REVIEW OF LITERATURE

Reliability engineering has developed through several decades of research involving probability theory, mathematical modelling, system analysis and maintenance optimisation.

The literature demonstrates a gradual movement from simple independent-component models towards more sophisticated approaches involving dependent failures, common-cause events and repair interactions.

2.1 Development of Classical Reliability Theory

Early reliability research established the mathematical foundation for analysing the probability of successful system operation. The work of Barlow and Proschan (1965) provided important theoretical contributions to reliability and life testing.

Their approach explained how probability distributions and component lifetime characteristics could be used to determine system reliability. The research established a systematic foundation for evaluating engineering components and systems.

Barlow and Proschan (1975) further developed the mathematical treatment of statistical theory in reliability and life testing. Their contributions remain relevant to the study of component failure distributions and system performance.

2.2 Reliability of Series Systems

Series-system reliability has received considerable attention because of its importance in industrial and technological applications.

For a series system, every essential component must remain operational. Consequently, the failure of any one component can cause complete system failure.

Rausand and Høyland (2004) presented comprehensive reliability modelling techniques for analysing engineering systems. Their work discussed component-level reliability, system structures, failure mechanisms and reliability evaluation.

Kumamoto and Henley (1996) also contributed significantly to probabilistic risk assessment, particularly regarding the modelling of complex system behaviour and uncertain failure events.

2.3 Correlated and Common-Cause Failures

Common-cause failures occur when a single underlying event causes multiple components to fail within the same system.

Examples include:

- Electrical supply disturbances affecting multiple devices.
- Extreme temperatures damaging several components.
- Flooding affecting interconnected equipment.
- Fire affecting multiple system units.
- Common software errors affecting redundant control systems.
- Improper maintenance affecting several components simultaneously.

2.4 Repairable Systems and Stochastic Modelling

Repairable-system reliability involves the study of systems that can return to operational condition following component failures.

Unlike non-repairable systems, repairable systems require the analysis of both failure and restoration processes.

2.5 Repair Dependency and Maintenance Constraints

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Repair dependency represents situations in which restoration activities are influenced by shared maintenance resources or operational conditions.

In practical environments, repair operations may be affected by:

- Shortage of spare parts.
- Shared diagnostic equipment.
- Maintenance scheduling constraints.
- Repair prioritisation policies.
- Dependence between component restoration procedures.

3. RESEARCH METHODOLOGY

3.1 Research Design

The present investigation adopts a quantitative, mathematical and analytical research design. The research is based on stochastic modelling and numerical simulation of a complex series system. A hypothetical engineering system is considered to demonstrate the mathematical relationships between component failures, common-cause events and repair activities.

The research consists of four major stages:

1. Development of a conventional series-system reliability model.
2. Incorporation of correlated component failure mechanisms.
3. Development of repair-dependent stochastic transitions.
4. Evaluation and comparison of reliability indicators.

3.2 Model Description

Consider a complex series system consisting of three essential components:

- Component A: Primary power supply.
- Component B: Control and monitoring unit.
- Component C: Main processing equipment.

All three components must operate simultaneously for the system to function successfully.

The system is therefore represented as:

$$S = A \times B \times C$$

3.3 Mathematical Formulation

3.3.1 Independent Component Reliability

For component i, the reliability function under a constant failure-rate assumption is:

$$[\, R_{i(t)} = e^{\{-\lambda_i t\}}]$$

For three independent components:

$$R_{s(t)} = R_{A(t)R_{B(t)R_{C(t)}]}$$

Therefore:

$$R_{s(t)} = e^{\{-(\lambda_A+\lambda_B+\lambda_C)t\}]}$$

This expression serves as the baseline model.

3.3.2 Incorporating Common-Cause Failures

Under a simplified model in which the common-cause event acts independently of the individual component failure processes:

$$R_{s(t)} = \; e^{\{-(\lambda_A+\lambda_B+\lambda_C+\lambda_c)t\}]}$$

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This formulation assumes that the common-cause event is catastrophic and causes system failure whenever it occurs.

3.3.3 Correlated Failure Representation

The failure dependence between two components can be represented through the joint probability:

$$P(X_i = 0, X_j = 0)$$

The statistical dependence may also be examined through a correlation coefficient:

$$\rho_{\{ij\}} = \{operatorname{Cov}\}(X_i, X_j)\}\left\{\sqrt{\{operatorname{Var}\}(X_i)operatorname{Var}\}(X_j)\}}\right\}]$$

A positive correlation indicates that component failure conditions tend to occur together more frequently than would be expected under independence, although the coefficient alone does not fully describe the joint failure mechanism.

3.3.4 Repair Mechanism

Under an independent repair assumption, the average repair time is:

$$MTTR_i = \frac{\{1\}}{\{\mu_i\}}$$

When repair resources are shared, the effective repair rate may depend on the number and condition of failed components.

3.4 Markov State Representation

The three-component system can be represented through eight possible operating configurations.

State	Component A	Component B	Component C	System condition
S0	Working	Working	Working	Operational
S1	Failed	Working	Working	Failed
S2	Working	Failed	Working	Failed
S3	Working	Working	Failed	Failed
S4	Failed	Failed	Working	Failed
S5	Failed	Working	Failed	Failed
S6	Working	Failed	Failed	Failed
S7	Failed	Failed	Failed	Failed

The system is operational only in state S0.

The transition structure incorporates:

- Individual component failure transitions.
- Common-cause transitions.
- Component repair transitions.
- Repair-resource restrictions.
- State-dependent repair intensities.

The transition-rate matrix is denoted by (Q).

The probability vector is:

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$$P(t) = [P_{0(t)}, P_{1(t)}, \dots, P_{7(t)}]$$

The state probability vector can be obtained through:

$$P(t) = P(0)e^{\{Qt\}}$$

provided the transition-rate matrix is correctly specified.

3.5 Reliability and Availability Measures

The research uses the following performance indicators.

3.6 Research Variables

The variables are divided into independent, dependent and moderating parameters.

Table 2. Classification of research variables

Variable category	Variables	Measurement
Independent variables	Individual failure rates	Failures per hour
Independent variables	Common-cause failure intensity	Events per hour
Independent variables	Repair rates	Repairs per hour
Independent variables	Repair congestion	Dimensionless parameter
Dependent variables	System reliability	Probability
Dependent variables	System availability	Probability
Dependent variables	Mean time to failure	Hours
Dependent variables	System unavailability	Probability
Modelling variables	Component operating states	Binary states
Modelling variables	Repair-resource availability	State-dependent parameter

3.7 Simulation and Analytical Procedure

The investigation follows a structured computational procedure.

Step 1: Define the system.

Three essential components are identified, and the series-system configuration is established.

Step 2: Specify failure rates.

Individual component failure intensities are assigned.

Step 3: Introduce correlated failure events.

Common-cause failure mechanisms are incorporated into the state-transition structure.

Step 4: Define repair dependencies.

The repair-resource adjustment function is applied to determine effective repair rates.

Step 5: Construct the transition matrix.

The eight-state Markov model is developed using the specified transition intensities.

Step 6: Calculate state probabilities.

The Kolmogorov equations are solved to determine the probability of each system condition.

Step 7: Estimate performance measures.

Reliability, availability, unavailability and mean time to failure are evaluated.

Step 8: Conduct comparative analysis.

The independent-failure model is compared with the correlated-failure and repair-dependent model.

3.9 Comparative Research Scenarios

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Three scenarios are proposed to evaluate the effects of different failure and repair assumptions.

Table 3. Proposed comparative modelling scenarios

Scenario	Failure assumption	Repair assumption	Purpose
Model I	Independent component failures	Independent repairs	Establish baseline performance
Model II	Correlated and common-cause failures	Independent repairs	Examine failure dependence
Model III	Correlated and common-cause failures	Dependent repairs	Examine combined effects

4. RESULTS AND INTERPRETATION

4.1 Introduction to Numerical Results

This section presents the numerical investigation of the proposed reliability model for a three-component complex series system. The analysis examines how independent failures, common-cause failures and repair dependencies influence overall system performance.

The following assumptions are maintained:

1. Each component has a constant individual failure intensity.
2. A common-cause event can cause all three components to fail simultaneously.
3. The system operates only when all three components are functional.
4. Repair times follow exponential distributions.

4.2 Analysis of Individual Component Reliability

The individual failure rates are:

$$[\lambda_A = 0.002]$$
$$[\lambda_B = 0.003]$$
$$[\lambda_C = 0.004]$$

The total individual failure intensity is:

$$[\lambda_s = 0.002 + 0.003 + 0.004]$$
$$\left\{\lambda_s = 0.009 \left\{\frac{failures}{hour}\right\}\right\}$$

For the independent-component model, system reliability is:

$$R_{s(t)} = e^{\{-0.009t\}]}$$

When a common-cause failure intensity of 0.001 failures per hour is introduced:

$$R_{c(t)} = e^{\{-(0.009+0.001)t\}]}$$
$$boxed\{R_{c(t)} = e^{\{-0.01t\}}\}$$

4.3 Comparison of System Reliability

Table 4. Comparison of system reliability under independent and correlated failure assumptions

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Operating time (hours)	Independent model	Model with common-cause failures
10	0.9139	0.9048
20	0.8353	0.8187
30	0.7634	0.7408
40	0.6977	0.6703
50	0.6376	0.6065
60	0.5827	0.5488
70	0.5326	0.4966
80	0.4868	0.4493
90	0.4449	0.4066
100	0.4066	0.3679

Interpretation

At 50 hours, the independent model produces a reliability estimate of approximately 0.6376, whereas the common-cause model produces approximately 0.6065.

The difference indicates that the introduction of an additional common-cause failure mechanism reduces the probability of successful operation throughout the specified period.

4.4 Analysis of Mean Time to Failure

For the independent model:

$$MTTF_I = \frac{\{1\}}{\{0.009\}]}$$

$$\left\{MTTF_I = \frac{111.11}{\{\ hours\}}\right\}$$

An important qualification is that repair activities do not change this first-failure MTTF under the present definition. Repairs influence subsequent operational availability and restoration, rather than preventing the first failure in this model.

4.5 Analysis of Steady-State System Availability

Steady-state availability represents the long-run probability that all essential components are operational.

It is calculated from the stationary distribution of the Markov process.

For the model with common-cause failures and independent repairs:

Table 5. Comparative analysis of steady-state system availability

Model	Failure mechanism	Repair mechanism	Availability
Model I	Independent failures	Independent repairs	93.94%
Model II	Common-cause and individual failures	Independent repairs	92.81%
Model III	Common-cause and individual failures	Dependent repairs	92.60%

Interpretation

The results indicate that the independent model achieves the highest steady-state availability. The introduction of common-cause failures reduces availability from approximately 93.94% to 92.81%.

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When repair-resource dependency is introduced, availability decreases further to approximately 92.60%.

Although the additional reduction due to repair dependency is relatively small under the selected parameter values, it demonstrates that restoration constraints can influence the long-term operational condition of a system.

4.6 Analysis of Repair Dependency

The repair congestion parameter \(\alpha\) determines how much the effective repair rate decreases when multiple components are failed.

The model uses:

$$\mu_i^* = \frac{\{\mu_i\}}{\{1 + \alpha(n_f - 1)\}}$$

where (n_f) represents the number of failed components.

Table 6. Sensitivity analysis of repair dependency

Repair congestion parameter	Steady-state availability	Unavailability
0.00	92.81%	7.19%
0.25	92.60%	7.40%
0.50	92.40%	7.60%
1.00	91.99%	8.01%

Interpretation

The sensitivity analysis demonstrates an inverse relationship between repair congestion and system availability.

When the congestion parameter increases, the effective repair rates decrease. Consequently, the system spends more time in failed configurations.

4.7 Graphical Representation of System Availability

The following figure presents the comparison of steady-state availability across the three principal modelling scenarios.

Steady-state availability under three modelling scenarios

Hypothetical Markov-model results; availability is expressed as a percentage.

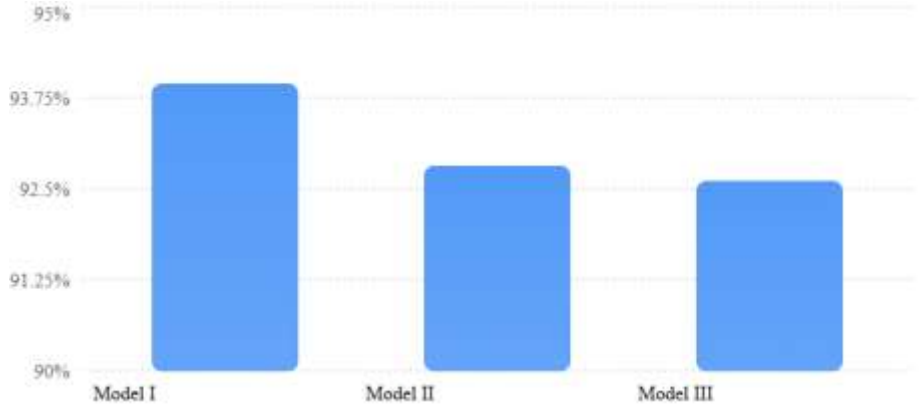


Figure 2. Comparison of steady-state availability under independent, common-cause and repair-dependent modelling assumptions.

The graphical comparison confirms that the independent model achieves the highest availability, while the combined failure-and-repair dependency model produces the lowest availability among the three scenarios.

The numerical investigation produces the following principal observations:

1. Common-cause failures reduce the probability of successful operation.
2. The introduction of common-cause failures reduces the hypothetical MTTF from 111.11 to 100 hours.
3. Repair-resource congestion reduces steady-state availability.
4. Systems with dependent failures and repairs require more comprehensive reliability models.
5. Maintenance-resource planning is an important element of overall system reliability improvement.

5. DISCUSSION AND CONCLUSION

5.1 Discussion

The present study investigates the reliability behaviour of a complex series system in which component failures and repair activities are not assumed to be completely independent.

The findings indicate that conventional reliability calculations may provide an incomplete representation of engineering system behaviour when common-cause failures are present.

The first important observation concerns the effect of common-cause failures. In the hypothetical numerical investigation, the inclusion of common-cause failure intensity reduced the estimated mean time to first system failure by 10%. This demonstrates the importance of identifying shared failure mechanisms when evaluating complex engineering systems.

The sensitivity analysis demonstrates that increasing repair congestion reduces the effective repair rate and consequently lowers system availability.

Another important finding concerns the relationship between system reliability and availability. Reliability represents the probability of performing the required function without failure throughout a specified period. Availability, in contrast, considers whether the system is operational at a particular time or over the long run, taking restoration into account.

The Markov approach provides a useful framework for analysing these operational conditions. By representing the system through multiple states, it becomes possible to examine the transitions between operational and failed configurations.

5.2 Practical Implications

The findings have several practical implications.

Industrial manufacturing: Production systems can improve operational continuity by identifying components that share electrical, mechanical or environmental failure mechanisms.

Power generation: Power plants can reduce common-cause risks through equipment segregation, independent backup arrangements and effective maintenance planning.

Communication systems: Network operators can improve reliability by reducing shared infrastructure vulnerabilities.

Transportation systems: Critical control and monitoring systems can benefit from dependency-aware reliability assessment.

Healthcare equipment: Hospitals can reduce equipment unavailability by improving spare-parts management and maintenance-resource coordination.

5.3 Recommendations

Based on the analytical findings, the following recommendations are proposed:

- Engineering organisations should identify common-cause failure mechanisms during system design and maintenance planning.
- Reliability assessments should consider both individual and dependent component failures.
- Critical spare parts should be maintained to reduce restoration delays.
- Maintenance personnel and diagnostic resources should be allocated according to component criticality.
- More advanced stochastic models should be adopted when exponential assumptions are unsuitable.

5.4 Conclusion

The present research develops a mathematical framework for evaluating the reliability of complex series systems under correlated failure and repair conditions.

The study demonstrates that the conventional assumption of independent component failures may not adequately represent complex engineering environments. Common-cause events can increase the probability of system failure and reduce the expected time to first failure. Similarly, shared repair resources and maintenance constraints can reduce system availability by extending the duration of failed states.

The hypothetical numerical results show that the introduction of common-cause failure mechanisms reduces mean time to first system failure from 111.11 hours to 100 hours. The corresponding steady-state availability decreases from approximately 93.94% under the baseline model to 92.81% under the common-cause model and 92.60% when repair dependency is also included.

The study concludes that a combined stochastic framework based on continuous-time Markov processes provides a useful foundation for analysing complex repairable series systems. It can assist engineers and maintenance planners in identifying operational weaknesses, evaluating restoration strategies and developing more effective reliability improvement programs.

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