

Analysis and Parametric Characterization of Alamouti STBC and QOSTBC Over Various Fading Channel

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Abstract — This Paper is mainly concerned with Orthogonal space-time block codes and their performances. The focus is on Bit error rate and channel estimation for wireless communication systems using Orthogonal space-time block code and Quasi Orthogonal space time block code. We first present the required background materials, discuss different implementations of space-time block codes using different numbers of transmit and receive antennas, and evaluate the performances of space-time block codes using binary phase-shift keying (BPSK), quadrature phase-shift keying (QPSK). Then, we investigate Orthogonal Space time block with Different channel like AWGN Channel, relaying Fading Channel and compare the bit error rate performance and maximum diversity gain.

Keywords- STBC, multiple input-multiple output (MIMO), AWGN, Rayleigh & Rician fading channel, QPSK, Bit Error Rate(BER).

INTRODUCTION

Most of the existing work in this area assumes that the antenna elements at the transmitter and the receiver of the MIMO system are placed far enough (spatially) such that the effect of the channel at a particular antenna element is different from the effect at all other antenna elements. This implies independent or spatially uncorrelated fading. This holds true only if spacing between transmit antennas or receive antennas is of the order of several wavelengths. However, if antenna spacing is not enough, the fading channel from multiple antennas might be correlated, and the performance will be degraded. Similarly, some of the existing

works assume quasi-static fading in which the channel characteristic remain constant over one frame period, while some others assume. (fully interleaved) fading in which channel varies from one symbol to another independently. However, one may not be able to interleave code words due to delays involved in interleaving. In that case, the channel will have temporal correlation.

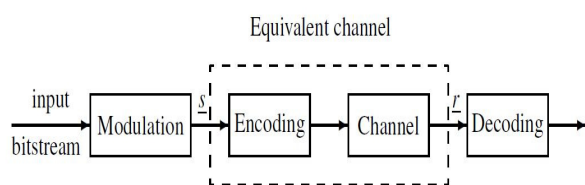
SPACE TIME BLOCK CODE

A code is mapping from the input bits to the transmitted symbols. Here, we assume that symbols are transmitted simultaneously from different antennas. Here, we study the performance of different codes by deriving some bounds on them. Then, we use the bounds to provide some guidance to design codes with “good” performance. Such guidance is called the design criterion. Most of the analyses in this paper are asymptotic analysis. Therefore, different asymptotic assumptions may result in different code criteria. We concentrate on a quasi-static Rayleigh fading wireless channel and some of the important design criteria that result in achieving maximum diversity and good performance at high SNRs.

A good code follows a design criterion that adds some notion of optimality to the code. In fact, the goal of defining a design criterion is to have a guideline for designing good codes. For example let us consider transmission over a binary symmetric channel using a linear binary block channel code. The bit error rate of the system depends on the Hamming distances of the codeword pairs. Defining the set of all possible codeword pairs and the

corresponding set of Hamming distances, we denote the minimum Hamming distance by d_{\min} . It can be shown that a code with minimum Hamming distance d_{\min} can correct all error patterns of weight less than or equal to $\lfloor (d_{\min} - 1)/2 \rfloor$, where

$\lfloor x \rfloor$ is the largest integer less than or equal to x . Therefore, for a given redundancy, a “good” code has a high minimum Hamming distance. The design criterion for such a code is to maximize the minimum possible Hamming distance among the codeword pairs. To compare two codes with similar redundancies, the one with higher Hamming distance is preferable. Similarly for an additive white Gaussian noise (AWGN) channel, a good design criterion is to maximize the minimum Euclidean distance among all possible codeword.



In what follows, we study the design criterion for space-time codes. We derive design criteria that guarantee the maximum possible diversity gain and coding gain at high SNRs. Also, we consider maximizing the mutual information between the input and output of the system as a design criterion.

QUASI-ORTHOGONAL SPACE TIME BLOCK CODE

In order to achieve the advantages of OSTBC schemes with properties close to such optimal codes providing full rate, the so called *quasi-orthogonal space-time block codes* (QOSTBC). These STBC were developed from quasi-orthogonal designs, where the orthogonality is relaxed to provide higher rate. QOSTBC allows a tradeoff between higher rate and maximum diversity. The full rate QOSTBC provides only half of the maximum diversity for four transmit antennas. The decoder of QOSTBC processes pairs

of transmitted symbols instead of a single symbol. Two maximum likelihood detectors are used in parallel to decode pairs of transmitted symbols in QOSTBC. The decision metric of is shown as the sum of two terms; thus minimizing the decision metric is equivalent to minimizing two terms independently. Two maximum likelihood detectors are used either in sequence or in parallel. Therefore, decoding pairs for QOSTBC is more complex than decoding single symbols for space-time block codes. This results in higher complexity decoding at the receiver. Specifically, the decoding complexity increases with the modulation level. In order to minimize the decision metric using maximum likelihood method, the receiver computes the decision metric over all possible symbols of a constellation or modulation level and decides in favor of the constellation symbols that minimize the decision metric. As the size of the constellation increases, the receiver must minimize the decision metric over large number of symbols. This will, subsequently, increase the transmission delay when high modulation schemes or more antennas are employed.

PAIR WISE DECODING

Full-rate orthogonal designs with complex elements in its transmission matrix are impossible for more than two transmit antennas. The only example of a full-rate full-diversity complex space-time block code using orthogonal designs is the Alamouti scheme. Here we rewrite the generator matrix for the Alamouti code to emphasize the indeterminate variables x_1 and x_2 in the design:

$$\mathcal{G}(x_1, x_2) = \begin{pmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{pmatrix}.$$

The main properties of an orthogonal design are simple separate decoding and full diversity. To design full-rate codes, we relax the simple separate decoding property. Here, we consider codes for which decoding pairs of symbols independently is possible. We call this class of codes quasi-orthogonal space-time block codes (QOSTBCs) for reasons that we discuss later.

First, let us consider the following QOSTBC :

$$\mathcal{G} = \begin{pmatrix} \mathcal{G}(x_1, x_2) & \mathcal{G}(x_3, x_4) \\ -\mathcal{G}^*(x_3, x_4) & \mathcal{G}^*(x_1, x_2) \end{pmatrix} = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & -x_4^* & x_1^* & x_2^* \\ x_4 & -x_3 & -x_2 & x_1 \end{pmatrix},$$

where a matrix G^* is the complex conjugate of G , for example

$$\mathcal{G}^*(x_1, x_2) = \mathcal{G}(x_1^*, x_2^*) = \begin{pmatrix} x_1^* & x_2^* \\ -x_2 & x_1 \end{pmatrix}.$$

We denote the i th column of G by V_i . For any indeterminate variables x_1, x_2, x_3, x_4 ,

We have

$$\langle V_1, V_2 \rangle = \langle V_1, V_3 \rangle = \langle V_2, V_4 \rangle = \langle V_3, V_4 \rangle = 0,$$

where $\langle V_i, V_j \rangle$ is the inner product of

vectors V_i and V_j . Therefore, the subspace created by V_1 and V_4 is orthogonal to the subspace created by V_2 and V_3 . This is the rationale behind the name “quasi-orthogonal” for the code. The minimum rank of the difference matrix $D(C_i, C_j)$ could be two. Therefore, following the definitions the diversity of the code is two, which is less than the maximum possible diversity of four. In the sequel, we show how to design full-diversity QOSTBCs.

The encoding for QOSTBCs is very similar to the encoding of orthogonal STBCs. To transmit b bits per time slot, we use constellations containing

$2b$ points. Using $4b$ bits, constellation symbols s_1, s_2, s_3, s_4 are selected. Setting $x_k = s_k$ for $k = 1, 2, 3,$

4 in the generator matrix G , we arrive at a codeword matrix $C = G(s_1, s_2, s_3, s_4)$. Then, at time t , the four elements in the t th row of C are transmitted from the four transmit antennas. Note that since four symbols s_1, s_2, s_3, s_4 are transmitted in four time slots, presents a rate one code. The orthogonality of the subspaces of the generator matrix results in the possibility of decoding pairs of symbols independently. Transmitting the data

symbols s_1, s_2, s_3, s_4 , we receive $r_{t,m}$ at time t and receive antenna m . Similar to decoding formulas the maximum-likelihood decoding for the QOSTBC

amounts to the following minimization problem: where C is derived by replacing x_k by s_k . Simple algebraic manipulation shows that the ML decoding amounts to minimizing the sum:

Popular QOSTBC Schemes

Jafarkhani Code

Jafarkhani proposed STBCs from quasi-orthogonal designs. For four antennas, a QOSTBC was constructed from the Alamouti scheme as follows

$$S_J = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ -s_3^* & -s_4^* & s_1^* & s_2^* \\ s_4 & -s_3 & -s_2 & s_1 \end{bmatrix},$$

and the corresponding equivalent channel matrix H_J is given by

$$H_J = \begin{bmatrix} h_1 & h_2 & h_3 & h_4 \\ -h_2^* & h_1^* & -h_4^* & h_3^* \\ -h_3^* & -h_4^* & h_1^* & h_2^* \\ h_4 & -h_3 & -h_2 & h_1 \end{bmatrix}.$$

Jafarkhani proposed different versions of the same code but our result is applicable to all versions with slight modifications. The Gramian matrix can be computed as

$$G_J = H_J^H H_J = \begin{bmatrix} \alpha & 0 & 0 & \beta \\ 0 & \alpha & -\beta & 0 \\ 0 & -\beta & \alpha & 0 \\ \beta & 0 & 0 & \alpha \end{bmatrix},$$

where $\alpha = |h_1|^2 + |h_2|^2 + |h_3|^2 + |h_4|^2$ and $\beta = 2 \operatorname{Re}(h_1 h_2^* - h_3 h_4^*)$. Notice that G_J has off diagonal terms which are interference terms, and this will affect the performance and lead to complex and computationally expensive decoding as well. Since the terms α and β are real, G_J is a real symmetric matrix (i.e., $G_J^T = G_J$).

Various Simulation Results

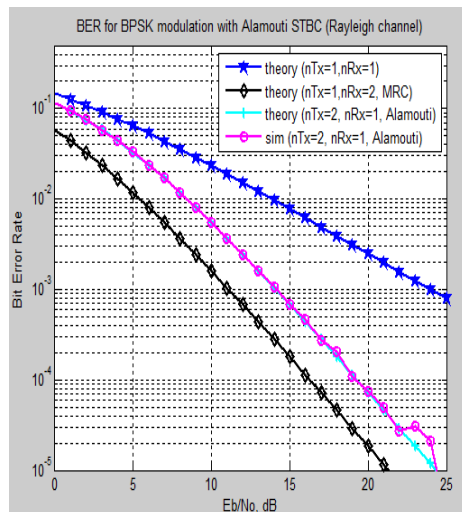


FIG 1: BER for BPSK modulation with Alamouti STBC for Rayleigh channel.

Here, The Result of Bit Error Rate Performance with Alamouti STBC over Rayleigh fading Channel is represented. Here, We have simulated the results for the different values of nTx and nRx .

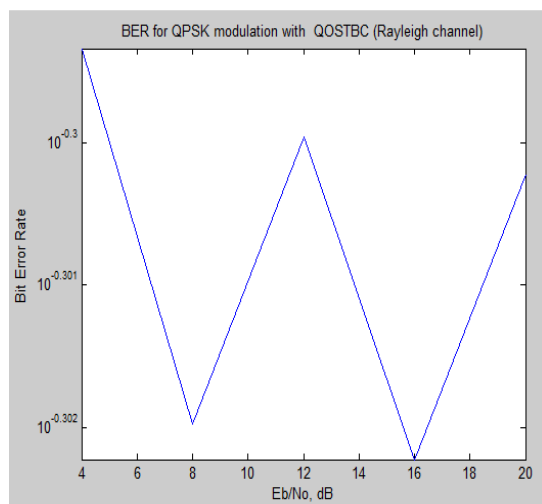


FIG 2: BER for QPSK modulation with QOSTBC for Rayleigh channel.

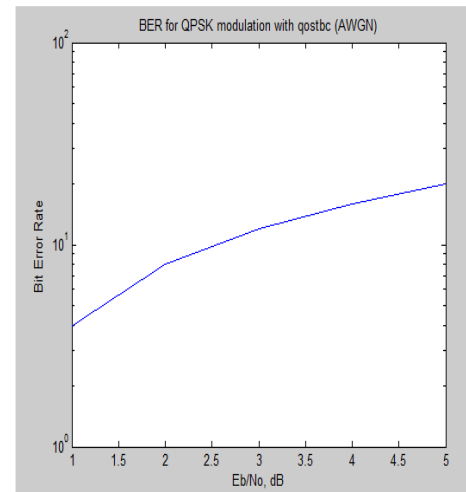


FIG 3: BER for QPSK modulation with QOSTBC for AWGN channel.

Here, In Fig 2 & Fig 3 We have depicted the various results for QOSTBC with QPSK modulation over various fading channel. Here, We have plotted the graph of BER V/s E_b/N_0 and analyze the response of QOSTBC.

CONCLUSION

Here, We have simulated the results for QOSTBC over various fading channel using QPSK modulation scheme. We have also simulated the results for Alamouti STBC using BPSK modulation over Rayleigh channel. From the results it is clear that there is vast change in BER response over Rayleigh channel & there is comparatively linear response over AWGN Channel.

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