Boundary Heat Flux Estimation in one-dimensional Heat Conduction Problem

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Abstract: **A numerical model is developed to estimate the boundary heat flux in 1D heat conduction problem using CGM. No prior information is used for the functional form of space wise varying heat flux. The energy equations are discredited using the finite volume method. The direct problem is first solved with a known heat flux at boundary and the temperature field of the domain is determined. Inverse method is then applied to predict this heat flux with some of the additional temperature data inside the solution domain obtained from the direct problem. The prediction of boundary heat flux by the present algorithm is found to be quite reasonable.**

Keywords: **CGM, heat flux, finite volume method.**

I. INTRODUCTION

When the direct measurements for a problem become difficult, inverse techniques are applied to estimate the boundary or inlet conditions, thermal properties or source term distributions of the material or medium, from the available experimental data. An inverse technique consists of typically both the forward model and an inverse model. In the forward model the effects (temperature profile of the body) can be found out on the basis of the causes (boundary temperature or flux). An inverse model is just the opposite of the forward model where the effects are known and the cause has to be found out.

The inverse heat conduction problem has received much attention since it has been widely used in practical engineering problems. Three types of inverse heat conduction problems are often raised namely estimation of surface conditions (boundary heat flux or temperature) [1-7], source term [8-21] and material thermal properties [22-27]. A number of deterministic optimization theories and algorithms have been developed toward the solution of these inverse problems. The CGM which is a deterministic method is mostly employed in inverse problems. The CGM is also called an iterative regularization method, which means the regularization procedure is performed during the iterative processes and thus the determination of optimal regularization conditions is not needed. The CGM derives from the perturbation principles and transforms the inverse problem to the solution of three problems, namely, the direct, sensitivity and the adjoint problem. Chen et al. [3] estimated the unknown heat flux of a pin fin base in two-dimensional problem by applying CGM with the adjoint equation. He presented that the effect of an initial guess value on estimating the heat flux with CGM is small. Huang and Wang [5] solved a three-dimensional (3-D) transient inverse heat conduction problem using the CGM and the general purpose commercial code CFX4.2-based inverse algorithm to estimate the unknown boundary heat flux in any 3-D irregular domain. Huang and Ozisik [8] used CGM to determine the unknown time wise variation of the strength of a plane surface heat source in one dimensional coordinates. Neto and Ozisik also applied CGM to estimate the time wise variation of the strength of single heat source [9] in 2D conduction medium and simultaneous estimation of time wise variation of the strength of two heat sources [10] in 1D conduction medium. However there are a few drawbacks with the CGM as they can converge to local minima and there is a requirement of initial guess value. CGM shows difficulty in estimation of sharp peaks or discontinuities on the functional variation of the source strength. Sawaf and Ozisik [22, 23] applied Levenberg–Marquardt (LM) algorithm to estimate linearly temperature dependent thermal conductivities of orthotropic solid.

Search-based methods or stochastic methods like genetic algorithm (GA) and differential evolution algorithm (DEA) have outstanding characteristics. These methods have advantages over gradient method for (i) non-requirement of initial guess and (ii) absence of complex formulations of sensitivity, adjoint and gradient equations that are inherently present in the gradient based methods. Moreover these search based methods converge to a global value. In recent years, stochastic methods have become a popular optimization tool for many areas of research. However, little work has been done in the inverse heat transfer problems. Raudensky et al. [24] studied the one dimensional inverse heat conduction problem for estimation of unknown material properties. Two artificial intelligence mechanisms, neural network and genetic algorithm, were applied in doing the inverse task. Both approaches can lead to a solution without stability problem. An inverse analysis based on an improved genetic algorithm to estimate an unknown transient heat source in one dimension heat conduction problem was presented by Liu [12]. Lobato et al. [13] used DEA for simultaneous estimation of source strength and position in one dimension conduction problem.

The main difference between the GA and DEA is the mutation scheme that makes DEA self adaptive. In DEA, all solutions have the same chance of being selected as parents without dependence of their fitness value. DEA employs a greedy selection process. The better one of new solution and its parent wins the competition providing significant advantage of converging performance over genetic algorithms. DEA mainly has three advantages; finding the true global minimum regardless of the initial parameter values, fast convergence, and using a few control parameters. Being simple, fast, easy to use, very easily adaptable for integer and discrete optimization, quite effective in nonlinear constraint optimization including penalty functions and useful for optimizing multimodal search spaces are the other important features of DEA.

The combined application of two or more methods called hybrid method has proved to be very powerful for solving IHTP. Chen and Chang [6] applied Laplace transform method and the finite element method in1D conduction problem and Chen et al. [7] applied Laplace transform method and the finite difference method in 2D conduction problem to estimate surface conditions.

The estimation of strength and position of internal heat source exhibits a practical strong appeal in areas such as chemical and mechanical engineering. Some of the available works in the literature determine either the location or the strength of a heat source [14]. There are also a few literatures where simultaneous estimation of location and strength [13, 15-21] of the source were determined. These studies are limited to conduction heat transfer only. Karami and Hematiyan [14] considered 2D heat conduction case to estimate the location or the strength of multiple point heat sources. Only one of these two variables was estimated, the other being known utilizing the boundary element method (BEM). Lobato et al. [13] applied DEA and Neto and Ozisik [15] applied CGM in one dimensional heat conduction problem while Khachfe and Jarny [16] applied the Finite element method (FEM) associated to CGM in 2D heat conduction problem to estimate the location and strength of point heat sources simultaneously. Lefèvre and Niliot [17-21] also estimated simultaneously the location and strength of heat sources in heat conduction problems by applying BEM.

II. CONCEPT OF INVERSE HEAT TRANSFER PROBLEM

The concept of IHTP is explained considering IHTP of boundary condition in 1D heat conduction problem which is one of the objectives of the present work. Figure 1.1 represents the schematic of 1D surface with boundary conditions. At time $t = 0$ the initial temperature of the surface is T_i. For time $t > 0$ a transient heat flux $q(t)$ is applied on the boundary at $x = 0$, while the boundary at $x = l$ is insulated.

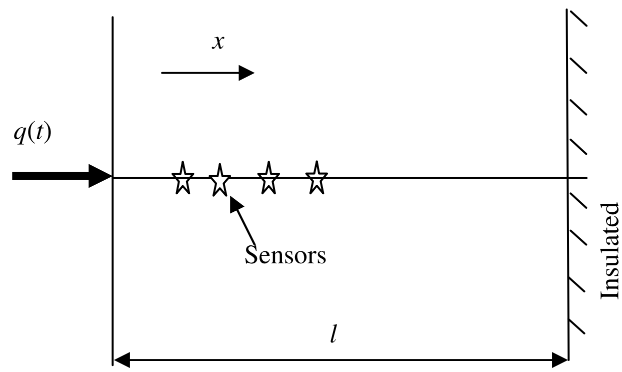

Fig. **Error! No text of specified style in document.**.1. Schematic of 1D heat conduction model.

The mathematical formulation of this heat conduction problem is given by:

$$k\frac{\partial^2 T(x,t)}{\partial x^2} = \rho c_p \frac{\partial T(x,t)}{\partial t} \quad \text{in } 0< x < l, \text{ for } t > 0 \qquad (1.1)$$

$$\frac{\partial T}{\partial x} = 0 \quad \text{at } x = l, \text{ for } t > 0 \qquad (1.2)$$

$$k\frac{\partial T}{\partial x} = -q(t) \quad \text{at } x = 0, \text{ for } t > 0 \qquad (1.3)$$

$$T = T_i \quad \text{in } 0 \le x \le l, \text{ for } t = 0 \qquad (1.4)$$

where, T is temperature distribution inside the surface of length l while k, ρ and c_p are thermal conductivity, density and specific heat respectively. Solution of the above model gives temperature profile $T(x,t)$ inside the surface. But in the real life situations, where it is not possible to measure the causal factors (surface temperature or boundary heat flux) directly, however the temperature data can be obtained by placing few sensors inside; a natural problem becomes to predict the unknown causal factor. This leads to the inverse heat transfer problem (IHTP).

Now taking the same example as above, the Eqs. (1.1-1.4) can be written in dimensionless form as:

$$\frac{\partial^2 \theta(X,\tau)}{\partial X^2} = \frac{\partial \theta(X,\tau)}{\partial \tau} \quad \text{in } 0< X < 1, \text{for } \tau > 0 \qquad (1.5)$$

$$\frac{\partial \theta}{\partial X} = 0 \quad \text{at } X = 1, \text{ for } \tau > 0 \qquad (1.6)$$

$$\frac{\partial \theta}{\partial X} = -Q(\tau) \quad \text{at } X = 0, \text{ for } \tau > 0 \qquad (1.7)$$

$$\theta = \theta_i \quad \text{in } 0 < X < 1, \text{ for } \tau = 0 \qquad (1.8)$$

where various dimensionless groups are defined as:

$$\theta = \frac{T}{T_{ref}}; X = \frac{x}{l}; \alpha = \frac{k}{\rho c_p}; \tau = \frac{\alpha t}{l^2}; Q = \frac{ql}{kT_{ref}}$$

where T_{ref} is reference temperature value and θ is dimensionless temperature distribution inside the surface of dimensionless length $L = 1.0$. To predict the unknown flux $Q(\tau)$, a quite natural hit and trial solution is - guess the flux and see how best the measured temperatures match with its direct problem solution profile. By the theory of statistical estimation, the optimal estimator (in general settings) is the one which minimizes the mean square norm of prediction errors and is called minimum mean square error estimator. Therefore the steps involved in the solution of an IHTP are as below:

(i)Start with an initial guess of the unknown flux.

(ii)Solve the direct heat equation to formulate the temperature profile.

(iii)Find the prediction error.

(iv)Minimize the mean square norm and hence optimize the guessed flux in a number of iterations.

From the above description of solution method, an IHTP gets converted to an optimization problem with objective as the minimization of the mean square norm of prediction errors and the constraint as the direct heat equation (1.5).

The temperature is measured by sensors at a number of locations ($X = X_m$), at discrete fixed time intervals. For the sake of analysis, the measurements are supposed to be continuous and a function of time $\theta_m(\tau)$. If $\theta_m(\tau)$ are free from measurement errors, then any guess boundary heat flux $Q^*(\tau)$, which satisfies the equation.

$$\theta_m(\tau)=\theta(X_m,\tau:Q^*(\tau)) \quad (1.9)$$

is the exact inverse solution. This implies that as the total mean square difference for all the sensors, given by

$$J(Q) = \sum_{m=1}^{M} \int_0^{\tau_f} [\theta(X_m,\tau;Q^*(\tau)) - \theta_m(\tau)]^2 d\tau \quad (1.10)$$

Decreases, the guessed flux $Q^*(\tau)$ approaches the exact solution $Q(\tau)$. Therefore $J(Q)$ is called the objective or performance function. Here M is the total number of measurements. Hence the inverse heat transfer problem becomes the following optimization problem

Minimize $J(Q)$

Subject to
$$\frac{\partial^2\theta}{\partial X^2} = \frac{\partial\theta}{\partial\tau} \quad (1.11)$$

To solve this optimization problem, any method of optimization can be used.

III. RESULT AND DISCUSSION

To examine the accuracy and computational efficiency of the algorithms for the estimation of the boundary heat flux in conduction problems different profiles of the heat flux, including a smooth function and a step function are selected. The effect of number of measurements M and error in measurements in 1D case on the accuracy of estimations is investigated.

The two different spacewise variations for the boundary heat flux of the following forms are considered.

$$Q_X(X) = 3000.0\sin(\pi(i-1)/100), \text{for } i=1,2,......,100 \quad (1.12)$$

$$Q(\tau) = \begin{cases} 3000.0, & \text{for } i=13,.....,15 \\ 0.0, & otherwise \end{cases} \quad (1.13)$$

where i is the indices in X direction. The energy equation, sensitivity equation and the adjoint equations are discredited using the FVM. The dimensionless length of slab is 1.0 and $\theta_i = 0$. The number of control volumes considered are 100 with step size $\Delta X = 0.01$.

The average percentage error is calculated as:

$$\%\text{ error} = \frac{1}{M}\sum_{i=1}^{M} \frac{Q_{ex}(x) - Q_{est}(x)}{Q_{ex}(x)} \text{ x } 100 \quad (1.14)$$

where Q_{ex} and Q_{est} are the exact and estimated heat flux values and D is the number of time steps. For the estimation, a single sensor is placed in the slab at $X = 4\Delta X$.

The effects of the number of measurements on the accuracy of the estimation, for sensors located at $X = 10\Delta X$ is investigated. Sensors are arranged with dimensionless spacing of $20\Delta Y$ between each two sensors. Similar arrangements are assumed for M = 19, 9 and 6 sensors with corresponding dimensionless spacing of $5\Delta Y$, $10\Delta Y$ and $15\Delta Y$, respectively. The test results shown in Fig 1.2 illustrate estimation of actual heat flux of triangular profile function with the number of transversal measurements M = 19, 9 and 6. Quantitatively, the average percentage error between actual and estimated temperature is 1.01%, 1.11% and 1.21%, with M = 19, 9 and 6 respectively. The average percentage error is calculated as:

$$\%\text{ error} = \frac{1}{M}\sum_{i=1}^{M} \frac{Q_{ex}(x) - Q_{est}(x)}{Q_{ex}(x)} \text{ x } 100 \quad (1.15)$$

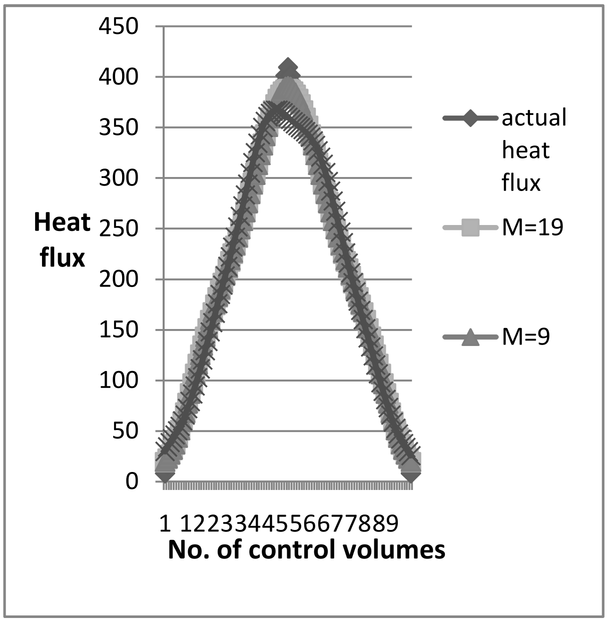

Fig. 1.2. Plots of estimated heat flux profile for a triangular input with variations in number of sensors embedded at transverse position.

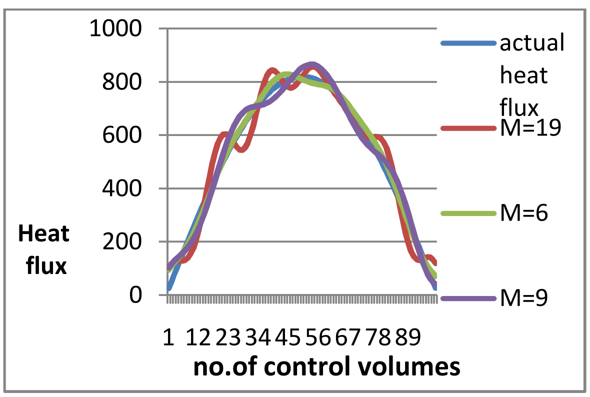

Fig. 1.3. Plots of estimated heat flux profile for a sine profile with variations in axial locations of thermocouples embedded.

Figure 1.3 refers to the case where all sensors are assumed to be embedded at the center line ($Y = 50\Delta Y$) along X direction. Six sensors ($M = 6$) are arranged corresponding to dimensionless spacing of $15\Delta X$. Similar arrangements are assumed with $M = 19$ and 9 corresponding to dimensionless spacing of $5\Delta X$ and $10\Delta X$, respectively. The results are shown in the same figure. The results show that at boundary the estimated profile deviates much from exact profile. The reason is that the heat flux originated at both regions is unlikely to diffuse into the centerline region given that the heat flux and flow fields are still developing. That is, there is almost no functional relationship between the measurement and estimated quantities. Therefore the estimations are not as good as compared to estimations of sensors embedded at transverse locations. This is also expected as the estimation is performed for a transverse heat flux profile and hence, the measurements taken in the same transverse direction are a better choice. With M = 19, the average percentage error between actual and estimated heat flux is 5%, while with M = 9 and 6, it is approximately 5.6%.

Figs.1.4 shows the result of step functional form with double discontinuity. It illustrates the effect of the number of measurements on the accuracy of the estimation, for sensor located at $X = 20\Delta X$. The number of transversal measurements considered here are M = 24 and 9 correspond to dimensionless spacing of $4\Delta Y$ and $10\Delta Y$, respectively. This case presents a very difficult case for an inverse analysis because the gradient of objective function is difficult to determine due to discontinuity present in the variation of the heat flux function. Even with twenty-four errorless measurements, the exact heat flux profile could not be fully recovered.

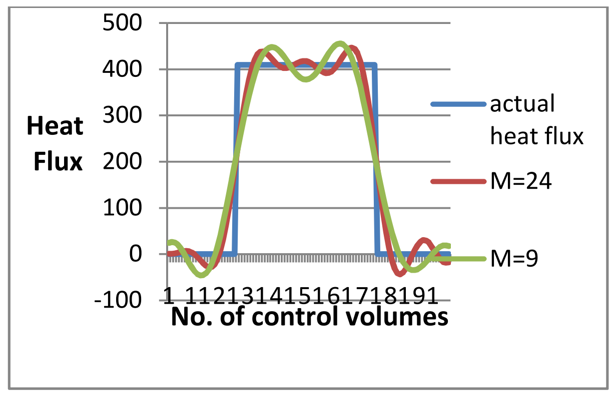

Fig. 1.4. Plots of estimated heat flux profile for a step input with variations in number of sensors embedded double discontinuity.

IV. CONCLUSION

The CGM was successfully applied for the solution of an inverse heat conduction problem to determine the unknown heat flux profile in a solid metal plate. The formulation of the inverse problem is presented in detail. Several test cases involving different profiles of heat flux distribution, different numbers and locations of sensors and measurement with artificial error are considered. Following are the conclusions:

(i)Four sensors (M = 4) are insufficient to produce good results to estimate heat flux while a single sensor could also give correct estimation of transient heat flux if the transient measurements are frequent.

(ii)The estimations of heat flux are not as good with sensors embedded along centerline as compared to sensors embedded at transverse locations.

(iii)With the errors in sensors included, higher numbers of measurements do not guarantee better prediction.

(iv)In general the accuracy of prediction improves when the sensors are located close to the boundary, whose description is to be predicted. However, if the sensors are kept very close to the boundary, where the gradients are high, the accuracy is found to be poor.

(v)The CGM is superior in terms of convergence speed in cases where measurements are available at each time step.

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