



Behaviour Analysis Of A Refrigeration Plant: A Case Study Of Saras Dairy Plant Jaipur – Rajasthan

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ABSTRACT

This paper presents a mathematical modeling and behaviour analysis of the refrigeration system at Saras Dairy Plant, Jaipur, Rajasthan. Focusing on three critical subunits—the condenser, expansion device, and evaporator—the study employs reliability engineering principles and the Regenerative Point Graphical Technique (RPGT) to quantify system performance. Analytical expressions are derived for mean time to system failure (MTSF), availability, server busy period, and expected maintenance visits, under varying failure and repair rates. Sensitivity analysis reveals the influence of subsystem reliability on plant uptime, maintenance workload, and cost optimization. The findings offer actionable insights for dairy plant managers to optimize preventive maintenance, minimize product losses, and enhance operational efficiency, contributing to improved food safety and economic sustainability.

Keywords: Reliability Analysis, Saras Dairy, MTSF, Availability, RPGT, Mathematical Modeling

1. Introduction:

In the global dairy industry, effective refrigeration is the linchpin of quality control and food safety. Dairy products are highly perishable, and even brief lapses in temperature management can lead to spoilage, financial losses, and health hazards. For large-scale facilities like Saras Dairy Plant in Jaipur, Rajasthan, the reliability of the refrigeration plant is paramount to ensuring uninterrupted processing, storage, and distribution of thousands of liters of milk and milk products daily. At the core of an industrial refrigeration system are its essential subunits: the condenser, expansion device, and evaporator. Each plays a distinct role—heat expulsion, refrigerant flow regulation, and heat absorption, respectively. Failure in any of these units can halt or degrade the cooling process, endangering product quality and resulting in costly downtime. The operational environment at Saras Dairy is particularly challenging, with Rajasthan's climatic extremes and fluctuating milk supply placing additional stress on refrigeration equipment. The plant's ability to maintain continuous chilling is crucial not only for product safety but also for compliance with food regulations and market demands for high-quality dairy products. Reliability engineering offers a framework for addressing these challenges by modeling the probability of system success or failure over time. In refrigeration, this approach supports informed decisions about preventive maintenance, equipment upgrades, and resource allocation. Cost optimization is equally critical: excessive maintenance inflates expenses, while insufficient attention increases the risk of breakdowns and wastage. Mathematical modeling, using tools such as Markov processes and RPGT, enables plant engineers to simulate operational scenarios, calculate performance indices like MTSF and availability, and perform sensitivity analyses. These techniques allow for the quantification of how changes in failure or repair rates impact system



behaviour, helping managers prioritize interventions where they yield the greatest benefit. This study develops a detailed reliability model of Saras Dairy's refrigeration plant, capturing the unique characteristics and failure/repair dynamics of the condenser, expansion device, and evaporator. By integrating sensitivity and cost analysis, the research provides a roadmap for optimizing plant performance, reducing maintenance costs, and safeguarding product integrity. The modeling approach and insights are not only valuable for Saras Dairy but are also broadly applicable to other industrial refrigeration systems facing similar reliability and operational challenges.

2. Scope of the Study:

- The study focuses on reliability and behaviour analysis of the refrigeration plant at Saras Dairy, Jaipur, considering the condenser, expansion device, and evaporator as primary subunits.
- It models failure and repair events as statistically independent and uses analytical techniques (Markov process, RPGT) for performance assessment.
- The research examines the impact of varying failure and repair rates on key performance indices, maintenance workload, and cost.
- Results are oriented to support decision-making in preventive maintenance, resource allocation, and operational strategy in dairy and other process industries.

3. Objectives:

- To develop a mathematical model for the refrigeration plant at Saras Dairy, quantifying reliability, availability, and mean time to system failure.
- To analyze the influence of failure and repair rates of critical subunits on system performance and maintenance demand.
- To perform sensitivity and cost optimization analysis, identifying optimal maintenance strategies for minimizing downtime and operational costs.
- To provide actionable recommendations for plant management to enhance operational efficiency, product safety, and economic sustainability.

4. Review of Literature

Arya & Verma (2025) demonstrated effective reliability and availability prediction using environmental modeling and simulation for embedded systems, emphasizing the value of data-driven maintenance. Agrawal et al. (2021) stressed the importance of performance analysis in water treatment plants, employing RPGT to inform maintenance scheduling and operational improvements. Dahiya et al. (2019) applied mathematical modeling to sugar industry crystallization systems, underscoring the versatility of such methods for complex process industries. Devi et al. (2018) explored path analysis for MTSF and availability in standby stochastic systems, providing insights directly relevant to modeling redundancy and failure in refrigeration plants. Kumar & Malik (2014) focused on computer systems, but their reliability modeling concepts—particularly the prioritization of hardware repair—translate well to industrial equipment. Kumar et al. (2018) and Rohit & Poonia (2023) used sensitivity analysis with RPGT to identify critical system parameters, revealing how targeted improvements in reliability and maintenance practices can yield significant gains in availability and performance. Ritikesh & Goel (2015) modeled availability for systems subject to degradation and repair, validating the application of RPGT in predicting downtime and maintenance

needs. Ranjan et al. (2015) contributed statistical approaches for competing risk models, broadening the analytical toolkit for reliability engineers.

5. System Description

The refrigeration system at Saras Dairy Plant comprises three key subunits: the condenser (E), expansion device (F), and evaporator (G), each with distinct failure and repair characteristics. The condenser expels heat but is prone to fouling and requires regular maintenance. The expansion device controls refrigerant flow, and malfunctions can disrupt the cooling process. The evaporator absorbs product heat and is susceptible to icing and fouling. The system is fully operational only when all units function optimally; failure of any unit or degraded performance in two units significantly reduces capacity. Independent failure and repair rates for each subunit are used for mathematical modeling to optimize maintenance and reliability strategies.

6. Assumptions and Notations

- The failure and repair events for each unit (condenser, expansion device, and evaporator) are statistically independent of each other.
- Transitioning between operational states (full capacity, reduced capacity, failed) is instantaneous and without additional failures or delays.
- **e, f, g:** Denote that the respective unit (E, F, G) is in a failed state.
- **$\bar{E}, \bar{F}, \bar{G}$:** Indicate a reduced capacity or degraded working state for the respective unit.
- $\lambda_1, \lambda_2, \lambda_3$: Failure rates for the Condenser (E), Expansion Device (F), and Evaporator (G) respectively.
- w_1, w_2, w_3 : Repair rates for the Condenser (E), Expansion Device (F), and Evaporator (G) respectively.

7. Transition Diagram: -

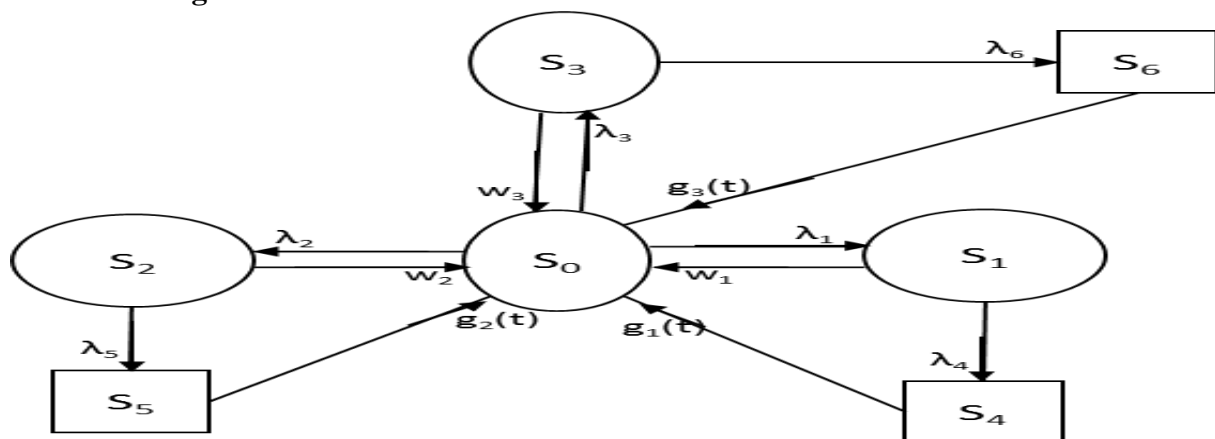


Figure 1: Transition Diagram

$S_0 = EFG$; $S_1 = EF\bar{g}$; $S_2 = E\bar{F}g$; $S_3 = EF\bar{G}$; $S_4 = Efg$; $S_5 = EfG$; $S_6 = EFg$

8. Transition Probabilities from the initial state ‘0’

$V_{0,0} = 1$ (Verified)

$V_{0,1} = (0,1) = p_{0,1} = (\lambda_1 / (\lambda_1 + \lambda_2 + \lambda_3))$

$V_{0,2} = (0,2) = p_{0,2} = (\lambda_2 / (\lambda_1 + \lambda_2 + \lambda_3))$

$V_{0,3} = \dots \dots$ Continue

9. Results and Discussions

MTSF(T₀): The states to which the system moves from initial state ‘0’, before going to any failed state are: ‘i’ = 0, 1, 2, 3 taking base state ‘ξ’ = ‘0’.

$$MTSF (T_0) = \left[\sum_{i,sr} \left\{ \frac{\left\{ pr \left(\xi \xrightarrow{sr(sff)} i \right) \right\} \mu_i}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[1 - \sum_{sr} \left\{ \frac{\left\{ pr \left(\xi \xrightarrow{sr(sff)} \xi \right) \right\}}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$T_0 = [(V_{0,0} \mu_0 + \{(\lambda_1/\lambda_1 + \lambda_2 + \lambda_3) \mu_1\} + \{(\lambda_2/\lambda_1 + \lambda_2 + \lambda_3) \mu_2\} + \{(\lambda_3/\lambda_1 + \lambda_2 + \lambda_3) \mu_3\}] / [1 - (0,1,0) + (0,2,0) + (0,3,0)]$$

Availability of the System: The states in which the system works in reduced or full are ‘j’ = 0, 1, 2, 3, taking base state ‘ξ’ = ‘0’ the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow j}) \right\} f_j \mu_j}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$A_0 = [\sum_j V_{\xi,j} f_j \mu_j] \div [\sum_i V_{\xi,i} f_i \mu_i^1] \dots \text{(Taking } \mu_i^1 = \mu_i \text{, as there is no waiting)}$$

Busy Period of the Server: The regenerative states where server ‘j’ = 1, 2, 3, 4, 5, 6 and regenerative states are ‘i’ = 0 to 6, taking base state ‘ξ’ = ‘0’, the total fraction of time for which the server remains busy is

$$B_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow j}) \right\} n_j}{\prod_{m_1 \neq \xi} \left\{ 1 - V_{m_1 m_1} \right\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\prod_{m_2 \neq \xi} \left\{ 1 - V_{m_2 m_2} \right\}} \right\} \right]$$

$$B_0 = [\sum_j V_{\xi,j} n_j] \div [\sum_i V_{\xi,i} \mu_i^1]$$

Expected Fractional Number of Inspections by the repair man: The regenerative states where the repair man does this job j = 1, 2, 3 Taking base state ‘ξ’ = ‘0’, the number of visit by the repair man is given by

$$V_0 = \left[\sum_{j,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow j}) \right\}}{\prod_{k_1 \neq \xi} \left\{ 1 - V_{k_1 k_1} \right\}} \right\} \right] \div \left[\sum_{i,sr} \left\{ \frac{\left\{ pr(\xi^{sr \rightarrow i}) \right\} \mu_i^1}{\prod_{k_2 \neq \xi} \left\{ 1 - V_{k_2 k_2} \right\}} \right\} \right]$$

$$V_0 = [\sum_j V_{\xi,j}] \div [\sum_i V_{\xi,i} \mu_i^1]$$

10. Particular Cases

Table 1: Mean time to System Failure (T₀)

T ₀	w = 0.92	w = 0.94	w = 0.96	w = 0.98	w = 1.00
λ = 0.44	2.585	2.585	2.585	2.585	2.585
λ = 0.46	1.264	1.264	1.264	1.264	1.264
λ = 0.48	0.941	0.941	0.941	0.941	0.941
λ = 0.50	0.789	0.789	0.789	0.789	0.789

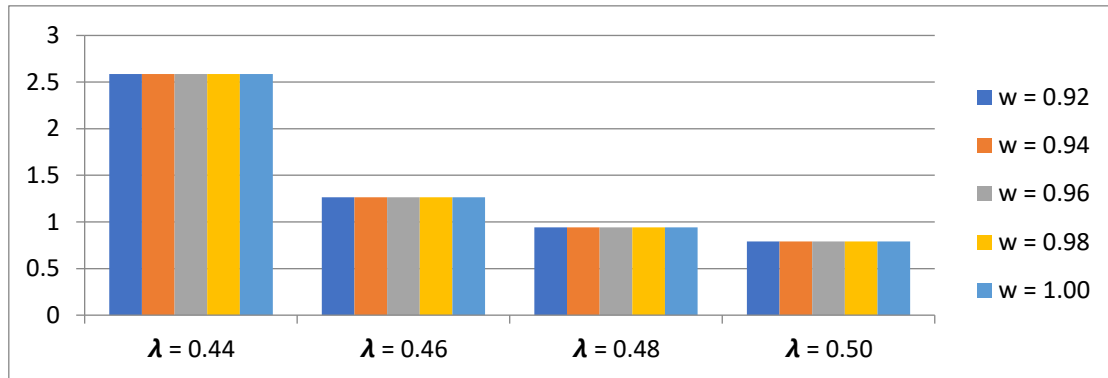


Figure 2: Mean time to System Failure (T0)

Table 1 presents the mean time to system failure (MTSF) for the refrigeration plant under different combinations of failure rates (λ) and repair rates (w). The data demonstrate that T_0 is primarily sensitive to the failure rate of the subsystems; as λ increases from 0.44 to 0.50, MTSF decreases sharply, indicating reduced operational lifetime. Interestingly, within each failure rate, varying the repair rate across the tested range has negligible impact on T_0 . This highlights that controlling and minimizing failure rates is more crucial for extending system life than increasing the speed of repairs alone.

Table 2: Availability of the System

A_0	w = 0.92	w = 0.94	w = 0.96	w = 0.98	w = 1.00
$\lambda = 0.44$	0.781	0.783	0.785	0.787	0.789
$\lambda = 0.46$	0.716	0.718	0.720	0.722	0.724
$\lambda = 0.48$	0.678	0.680	0.682	0.684	0.686
$\lambda = 0.50$	0.612	0.614	0.616	0.618	0.620

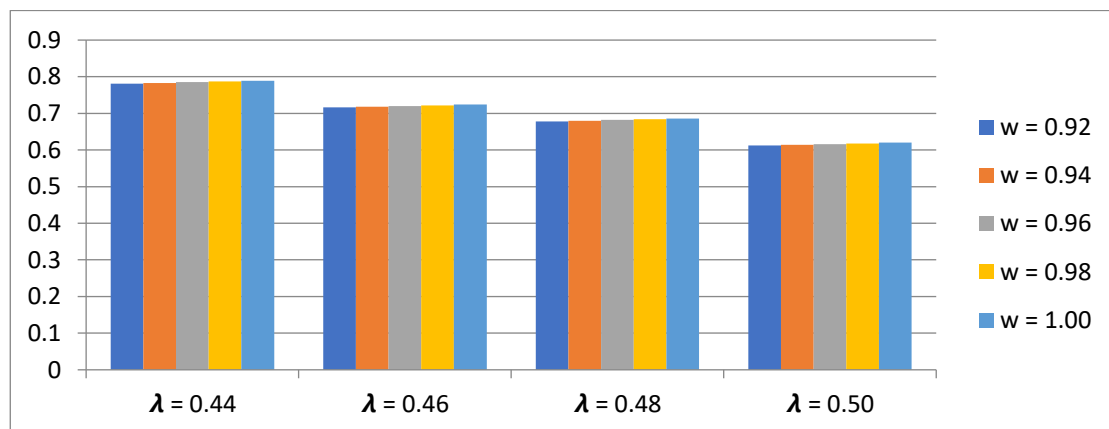


Figure 3: Availability of the System

Table 2 details the steady-state availability (A_0) of the refrigeration plant as a function of both repair (w) and failure (λ) rates. The results show that availability improves slightly with increasing repair rates for a fixed failure rate, but declines substantially as the failure rate increases. For example, at $\lambda = 0.44$, A_0 rises from 0.781 to 0.789 as w increases from 0.92 to 1.00, while a higher failure rate ($\lambda = 0.50$) results in a much lower availability overall. This underscores the importance of both reducing failures and optimizing repair efficiency to maximize plant uptime.

Table 3: Server of the Busy Period (B_0)

B_0	$w = 0.92$	$w = 0.94$	$w = 0.96$	$w = 0.98$	$w = 1.00$
$\lambda = 0.44$	0.313	0.312	0.311	0.310	0.309
$\lambda = 0.46$	0.258	0.257	0.256	0.255	0.254
$\lambda = 0.48$	0.184	0.183	0.182	0.181	0.180
$\lambda = 0.50$	0.056	0.055	0.054	0.053	0.052

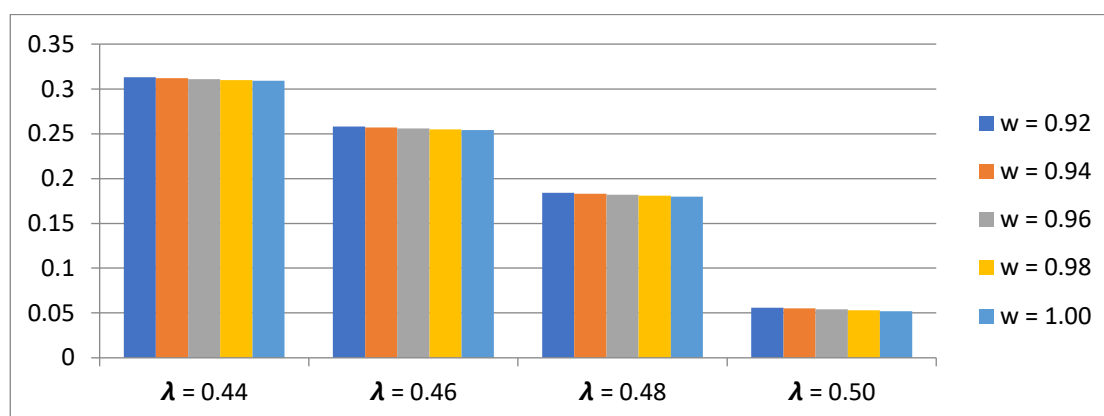


Figure 4: Server of the Busy Period (B_0)

Table 3 reports the server busy period (B_0), representing the fraction of time the maintenance server is actively engaged in repairs. The data indicate that B_0 decreases with increasing repair rates for any given failure rate, meaning that faster repairs lighten the server's workload. Conversely, higher failure rates lead to a greater proportion of time spent on repairs, highlighting the need for preventive maintenance and reliability improvements to keep maintenance demands manageable.

Table 4: Expected Fractional Number of Inspections by the Repairman (V_0)

V_0	$w = 0.92$	$w = 0.94$	$w = 0.96$	$w = 0.98$	$w = 1.00$
$\lambda = 0.44$	0.525	0.524	0.523	0.522	0.521
$\lambda = 0.46$	0.524	0.523	0.522	0.521	0.520
$\lambda = 0.48$	0.523	0.522	0.521	0.520	0.519
$\lambda = 0.50$	0.522	0.521	0.520	0.519	0.518

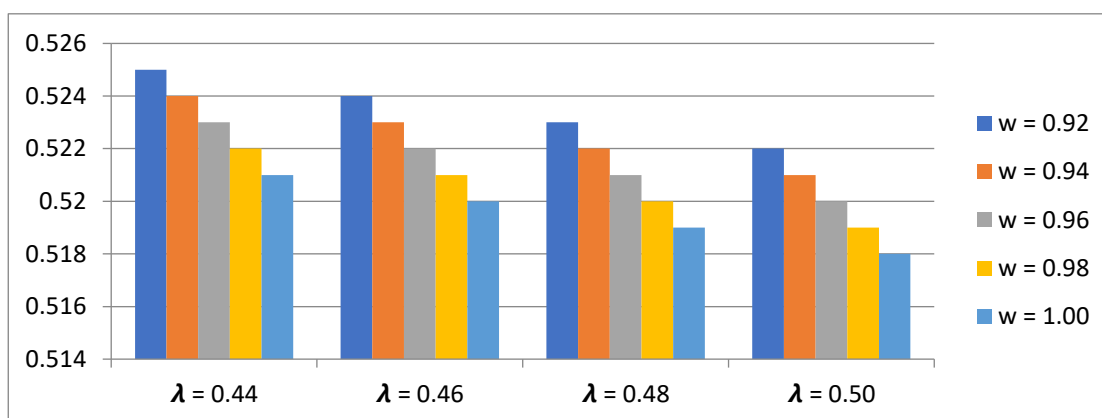


Figure 5: Expected Fractional Number of Inspections by the Repairman (V_0)



Table 4 quantifies the expected fractional number of inspections or interventions (V_0) performed by the repairman under varying failure and repair rates. V_0 remains relatively stable across repair rates but shows a slight decrease as repair rates improve, reflecting more efficient maintenance cycles. As failure rates increase, V_0 remains high, emphasizing that elevated equipment failures require sustained inspection and intervention efforts from maintenance personnel.

11. Conclusion

The mathematical modeling and behaviour analysis of the Saras Dairy refrigeration plant reveal that the reliability and longevity of the system are most strongly influenced by the failure rates of its key subunits. While enhancing repair rates does marginally improve availability and reduce server workload, the most significant gains are achieved through proactive failure prevention strategies. The study demonstrates that prioritizing preventive maintenance, high-quality component selection, and robust operational protocols can minimize unplanned downtime, optimize maintenance resources, and ensure consistent product quality. The analytical framework and sensitivity analysis presented here provide plant managers with actionable insights for data-driven decision-making and operational optimization.

12. Future Scope

- **Model Expansion:** Future work can extend the model to include more complex system architectures, such as additional subsystems or redundant pathways, to better reflect real-world plant configurations.
- **Non-Exponential Distributions:** Incorporating non-exponential failure and repair time distributions would enhance the realism of the analysis, enabling more accurate predictions for systems with varying operational environments.
- **Economic Analysis:** Integrating detailed cost-benefit analysis or profit optimization could directly link reliability improvements to financial outcomes and guide strategic investments.
- **Environmental Factors:** Future studies could assess the impact of external factors such as ambient temperature, humidity, and energy supply fluctuations on refrigeration system reliability.

13. Limitations

- **Simplified Assumptions:** The analysis assumes statistically independent failure and repair events and primarily exponential time distributions, which may not fully capture all real-world dependencies and complexities.
- **Fixed Rates:** Failure and repair rates are modeled as constant, potentially overlooking time-dependent wear, aging, or maintenance learning effects.
- **No Human or Supply Delays:** The model does not account for delays related to human error, supply chain disruptions, or logistical constraints, which can affect real-world repair and maintenance efficiency.
- **Limited Subsystems:** Only three critical subunits are considered; auxiliary or support equipment failures are not modeled, which may also impact system availability.
- **Focus on Technical Metrics:** The study emphasizes technical reliability indices and does not directly address broader operational issues such as workforce training, regulatory compliance, or customer impact.



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