

Review Paper of Binary Codes for Error Detection and Correction

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Abstract— Golay Code is a type of Error Correction code discovered and performance very close to Shannon's limit. Good error correcting performance enables reliable communication. Since its discovery by Marcel J.E. Golay there is more research going on for its efficient construction and implementation. The binary Golay code (G23) is represented as (23, 12, 7), while the extended binary Golay code (G24) is as (24, 12, 8). High speed with low-latency architecture has been designed and implemented in Virtex-4 FPGA for Golay encoder without incorporating linear feedback shift register. This brief also presents an optimized and low-complexity decoding architecture for extended binary Golay code (24, 12, 8) based on an incomplete maximum likelihood decoding scheme.

Index Terms— Golay Code, Decoder, Encoder, Field Programmable Gate Array (FPGA)

I. INTRODUCTION

Communication system transmits data from source to transmitter through a channel or medium such as wired or wireless. The reliability of received data depends on the channel medium and external noise and this noise creates interference to the signal and introduces errors in transmitted data. Shannon through his coding theorem showed that reliable transmission could be achieved only if data rate is less than that of channel capacity. The theorem shows that a sequence of codes of rate less than the channel capacity have the capability as the code length goes to infinity [1]. Error detection and correction can be achieved by adding redundant symbols to the original data called as error correction and correction codes (ECCs). Without ECCs data need to retransmit if it could detect there is an error in the received data. ECC are also called as for error correction (FEC) as we can correct bits without retransmission. Retransmission adds delay, cost and wastes system throughput. ECCs is really helpful for long distance one way communications such as deep space communication or satellite communication. They also have application in wireless communication and storage devices [2].

Error detection and correction helps in transmitting data in a noisy channel to transmit data without errors. Error detection refers to detect errors if any received by the receiver and correction is to correct errors received by the receiver. Different errors correcting codes are there and can be used depending on the properties of the system and the application in which the error correcting is to be introduced. Generally

error correcting codes have been classified into block codes, convolutional codes, LDPC Code and Goley Code [3, 4].

Error-Detecting codes

Whenever a message is transmitted, it may get scrambled by noise or data may get corrupted. To avoid this, we use error-detecting codes which are additional data added to a given digital message to help us detect if an error occurred during transmission of the message. A simple example of error-detecting code is parity check [5].

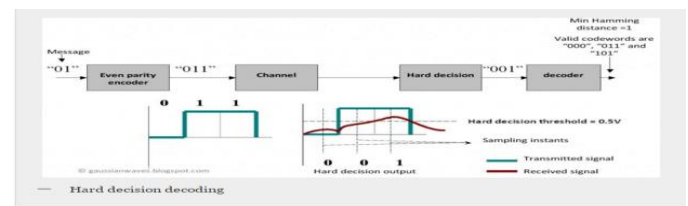


Figure 1: Block Diagram of Error Detecting Codes

Error-Correcting codes:-

Along with error-detecting code, we can also pass some data to figure out the original message from the corrupt message that we received. This type of code is called an error-correcting code. Error-correcting codes also deploy the same strategy as error-detecting codes but additionally, such codes also detect the exact location of the corrupt bit [6].

In error-correcting codes, parity check has a simple way to detect errors along with a sophisticated mechanism to determine the corrupt bit location. Once the corrupt bit is located, its value is reverted (from 0 to 1 or 1 to 0) to get the original message [7].

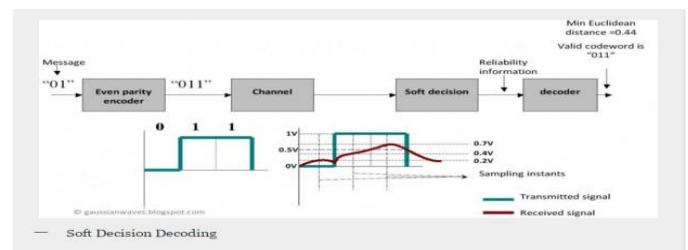


Figure 2: Block Diagram of Error Correction Code

Voltage levels of the received signal at each sampling instant are shown in the figure. The soft decision block calculates the Euclidean distance between the received signal and the all possible codewords [8].

The minimum Euclidean distance is "0.49" corresponding to "0 1 1" codeword (which is what we transmitted). The decoder selects this codeword as the output. Even though the parity

encoder cannot correct errors, the soft decision scheme helped in recovering the data in this case. This fact delineates the improvement that will be seen when this soft decision scheme is used in combination with forward error correcting (FEC) schemes like convolution codes, Goley Code etc.

II. LITERATURE REVIEW

Pallavi Bhoyar et al. [1], this paper is based on cyclic redundancy check based encoding scheme. High throughput and high speed hardware for Golay code encoder and decoder could be useful in digital communication system. In this paper, a new algorithm has been proposed for CRC based encoding scheme, which devoid of any linear feedback shift registers (LFSR). In addition, efficient architectures have been proposed for both Golay encoder and decoder, which outperform the existing architectures in terms of speed and throughput. The proposed architecture implemented in virtex-4 Xilinx power estimator. Although the CRC encoder and decoder is intuitive and easy to implement, and to reduce the huge hardware complexity required. The proposed method it improve the transmission system performance level. In this architecture our work is to design a Golay code based on encoder and decoder architecture using CRC generation technique. This technique is used to reduce the circuit complexity for data transmission and reception process.

Pedro Reviriego et al. [2], Memories that operate in harsh environments, like for example space, suffer a significant number of errors. The error correction codes (ECCs) are routinely used to ensure that those errors do not cause data corruption. However, ECCs introduces overheads both in terms of memory bits and decoding time that limit speed. In particular, this is an issue for applications that require strong error correction capabilities. A number of recent works have proposed advanced ECCs, such as orthogonal Latin squares or difference set codes that can be decoded with relatively low delay. The price paid for the low decoding time is that in most cases, the codes are not optimal in terms of memory overhead and require more parity check bits. On the other hand, codes like the (24,12) Golay code that minimize the number of parity check bits have a more complex decoding. A compromise solution has been recently explored for Bose–Chaudhuri–Hocquenghem codes.

Satyabrata Sarangi et al. [3], this brief lays out cyclic redundancy check-based encoding scheme and presents an efficient implementation of the encoding algorithm in field programmable gate array (FPGA) prototype for both the binary Golay code (G23) and extended binary Golay code (G24). High speed with low-latency architecture has been designed and implemented in Virtex-4 FPGA for Golay encoder without incorporating linear feedback shift register. This brief also presents an optimized and low-complexity decoding architecture for extended binary Golay code (24, 12, 8) based on an incomplete maximum likelihood decoding scheme. The proposed architecture for decoder occupies less

area and has lower latency than some of the recent work published in this area. The encoder module runs at 238.575 MHz, while the proposed architecture for decoder has an operating clock frequency of 195.028 MHz. The proposed hardware modules may be a good candidate for forward error correction in communication link, which demands a high-speed system.

P. Adde et al. [4], J.H. Conway and N.J.A. Sloane have introduced an algorithm for the exact maximum-likelihood decoding of the Golay (24, 12) code in the additive white Gaussian noise channel that requires significantly fewer computations than previous algorithms. An efficient bit-serial VLSI implementation of this algorithm is described. The design consists of two chips developed using path programmable logic (PPL) and an associated system of automated design tools for three- μ m NMOS technology. It is estimated that this decoder will produce an information bit every 1.6-2.4 μ s. Higher speeds can be achieved by using a faster technology or by replicating the chips to perform more operations in parallel.

T.-C. Lin et al. [5], new simple encoding and trellis decoding techniques for Golay codes, based on generalised array codes (GACs) are proposed. The techniques allow the design of (23, 12, 7) Golay and (24, 12, 8) extended Golay codes with minimal trellises. It is shown that these trellises differ only in the last trellis depth with different labelling digits.

Xiao-Hong Peng et al. [6], Two product array codes are used to construct the (24, 12, 8) binary Golay code through the direct sum operation. This construction provides a systematic way to find proper (8, 4, 4) linear block component codes for generating the Golay code, and it generates and extends previously existing methods that use a similar construction framework. The code constructed is simple to decode.

The two array codes concerned are both two-dimensional product codes. Where G_1 is the 4×8 generator matrix of C_1 , and '0' represents a 4×8 null matrix. It is noted that the (24, 8, 8) code is also used in [13] for constructing the (24, 12, 8) code. However, the construction presented in this reference leads to a nonlinear (24, 12) code, since one of the other component codes, the (8, 3) code, used in the construction is nonlinear. No direct proof is given for showing that the distance of the code is 8, although its weight distribution turns out to be that of the extended Golay code.

Table 1: Comparison of the Encoder architecture considering latency and area

Reference	Number of Slice	Slice Flip Flop	LUTs
[1]	85	80	149
[2]	97	89	158

[3]	104	94	172
[4]	112	104	201
[5]	156	112	232
[6]	189	118	273

III. BINARY CODES

Block codes are referred to as (n, k) codes. A block of k information bits are coded to become a block of n bits. $n=k+r$, where r is the number of parity bits and k is the number of information bits.

The more commonly employed Block codes are:

1. Single Parity-Check Bit Code
2. Repeated Codes
3. Hadamard Code
4. Hamming Code
5. Convolution Code Codes
6. Cyclic Codes
7. Golay Code
8. Extended Golay Codes

Marcel Golay was born in Neuchatel, Switzerland in 1902. He was a successful mathematician and information theorist who was better known for his contribution to real world applications of mathematics than any theoretical work he may have done [9, 10]. Golay's sought the perfect code. Perfect codes are considered the best codes and are of much interest to mathematicians. They play an important role in coding theory for theoretical and practical reasons. The following is a definition of a perfect code:

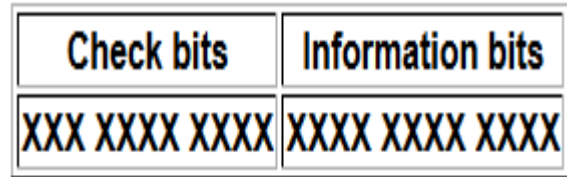
A code C consisting of N codewords of length N containing letters from an alphabet of length q , where the minimum distance $d=2e+1$ is said to be perfect if:

$$\sum_{i=0}^e \binom{n}{i} (q-1)^i = \frac{q^n}{N} \quad (1)$$

There are two closely related binary Golay codes. The extended binary Golay code G_{24} encodes 12 bits of data in a 24-bit word in such a way that any 3-bit errors can be corrected or any 7-bit errors can be detected. The other, the perfect binary Golay code G_{23} has codewords of length 23 and is obtained from the extended binary Golay code by deleting one co-ordinate position. In standard code notation the codes have parameters $[24, 12, 8]$ and $[23, 12, 7]$ corresponding to the length of the codewords, the dimension of the code and the minimum Hamming distance between two codewords respectively. In mathematical terms, the extended binary Golay code, G_{24} consists of a 12-dimensional subspace W of the space $V=F^{224}$ of 24-bit words such that any two distinct elements of W differ in at least eight coordinates. By linearity, the distance statement is equivalent to any non-zero element of W having at least eight non-zero coordinates. The

possible sets of non-zero coordinates as w ranges over W are called code words. In the extended binary Golay code, all code words have the Hamming weights of 0, 8, 12, 16, or 24. Up to relabeling coordinates, W is unique. The perfect binary Golay code, G_{23} is a perfect code. That is the spheres of radius three around code words form a partition of the vector space.

Codeword Structure: A codeword is formed by taking 12 information bits and appending 11 check bits which are derived from a modulo-2 division, as with the CRC. Golay $[23, 12]$ Codeword. The common notation for this structure is Golay $[23, 12]$, indicating that the code has 23 total bits, 12 information bits, and $23-12=11$ check bits. Since each codeword is 23 bits long, there are 2^{23} , or 8,388,608 possible binary values. However, since each of the 12-bit information fields has only one corresponding set of 11 check bits, there are only 212, or 4096, valid Golay code words.



Golay [24,12] Codeword		
Parity bit	Check bits	Information bits
X	XXX XXXX XXXX	XXXX XXXX XXXX

Figure 3: Block Diagram of Golay Code

The binary Golay code leads us to the extended Golay code. Codes can be easily extended by adding an overall parity check to the end of each code word.

This extended Golay Code can be generated by the 12×24 matrix $G = [I_{12} \mid A]$, where I_{12} is the 12×12 identity matrix and A is the 12×12 matrix

$$A = \begin{bmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

The binary linear code with generator matrix G is called the extended binary Golay code and will be denoted by G_{24} .

The extended Golay code has a minimum distance of 8. Unlike the (23, 12) code, the extended Golay code is not perfect, but simply quasi perfect.

Properties of the extended binary Golay code

- The length of G_{24} is 24 and its dimension is 12.
- A parity-check matrix for G_{24} is the 12×24 matrix $H = [A \mid I_{12}]$.
- The code G_{24} is self-dual, i.e., $G \perp 24 = G_{24}$.
- Another parity-check matrix for G_{24} is the 12×24 matrix $H_0 = [I_{12} \mid A] (= G)$.
- Another generator matrix for G_{24} is the 12×24 matrix $G_0 = [A \mid I_{12}] (= H)$.
- The weight of every codeword in G_{24} is a multiple of 4.
- The code G_{24} has no codeword of weight 4, so the minimum distance of G_{24} is $d = 8$.
- The code G_{24} is an exactly three-error-correcting code.

Table 2: Comparison Result of Different Types of Binary Code

Binary Code	Error Detection	Error Correction	Efficiency	Distance
Single Parity-Check Bit Code	1	1	80-85%	low
Hadamard Code	2	1	50%	Low
Hamming Code	2	1	85-90%	low
Convolution Code	2	2	85-90%	Medium
Cyclic Codes	3	2	75-80%	Medium
Golay Codes	7	3	70-75%	Large
Extended Golay Codes	8	3	80-85%	Very Large

IV. METHODOLOGY

In each step during polynomial division, simply binary XOR operation occurs for modulo-2 subtraction. The residual result obtained at each step during the division process is circularly left shifted by number of leading zeros present in the result. A 12:4 priority encoder is used to detect efficiently the number of leading zeros before first 1 bit in the residual result in each step. A circular shift register is used to shift the intermediate result by the output of priority encoder. A 2:1 multiplexer is used to select the initial message or the circularly shifted intermediate result. The control signal used for the multiplexer and the controlled sub-tractor is denoted as p , which is bit wise OR operation of priority encoder output. A controlled sub-tractor is used for loop control mechanism. Initially, one input of sub-tractor is initialized with 11, which is the number of zeros appended in the first step of the long division process and it gets updated with the content of $R7$ register due to multiplexer selection after each iteration. The output of the priority encoder is the other input to the sub-tractor. After the final iteration, the result of sub-tractor is zero, which is stored in register $R7$. The register $R6$ is loaded when the content of register $R7$ becomes zero, which depicts the end of the division process and hence the check bits generation process.

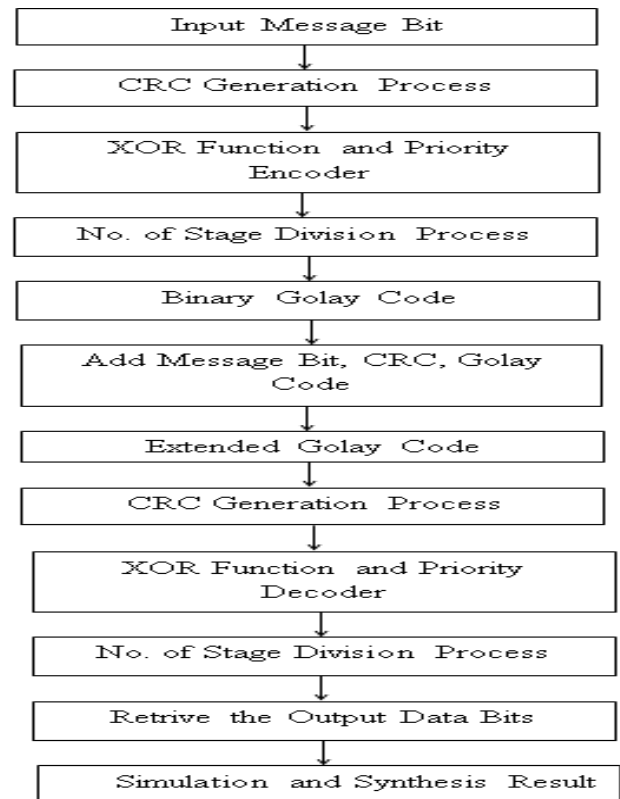


Figure 4: Flow Chart of Golay Encoder and Decoder

V. CONCLUSION AND FUTURE SCOPE

In this paper, the Golay code and operation for various encoder and decoder is discussed. This encoding and decoding algorithm have been successfully applied to short block codes such as Golay code. Decoding algorithm consists of syndrome measurement unit, weight measurement unit and weight constraint.

The purpose of this study is to review the published encoding and decoding models in the literature and to critique their reliability effects. We will try to reduce the area, maximum combinational path delay (MCPD) of decoding algorithm of Golay code.

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